# Lazy Robots and Traveling Guards

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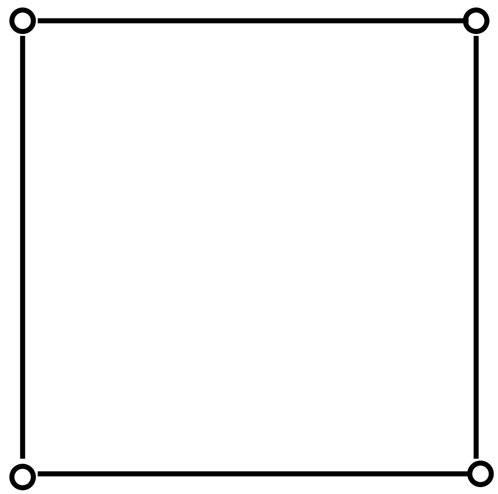
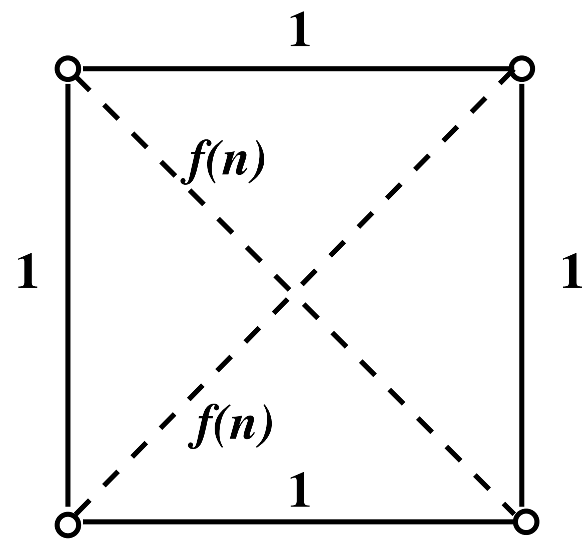
# The TSP is hard.

– Barbie

# TSP is not Approximable to any Polynomial Bound

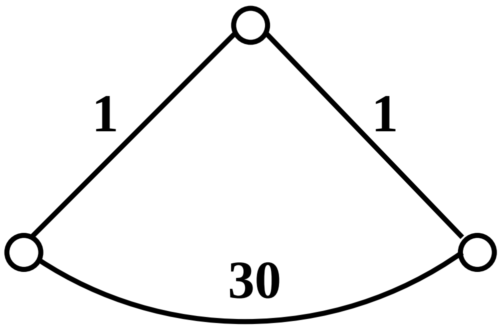




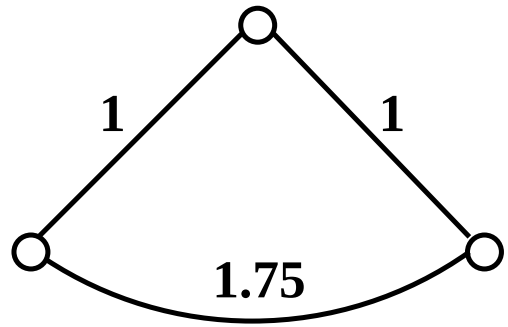

Define Hamiltonian cycle.

If $f(n)$-approximable, finding a tour on right finds a Hamiltonian cycle on left.

# Euclidean TSP




General TSP

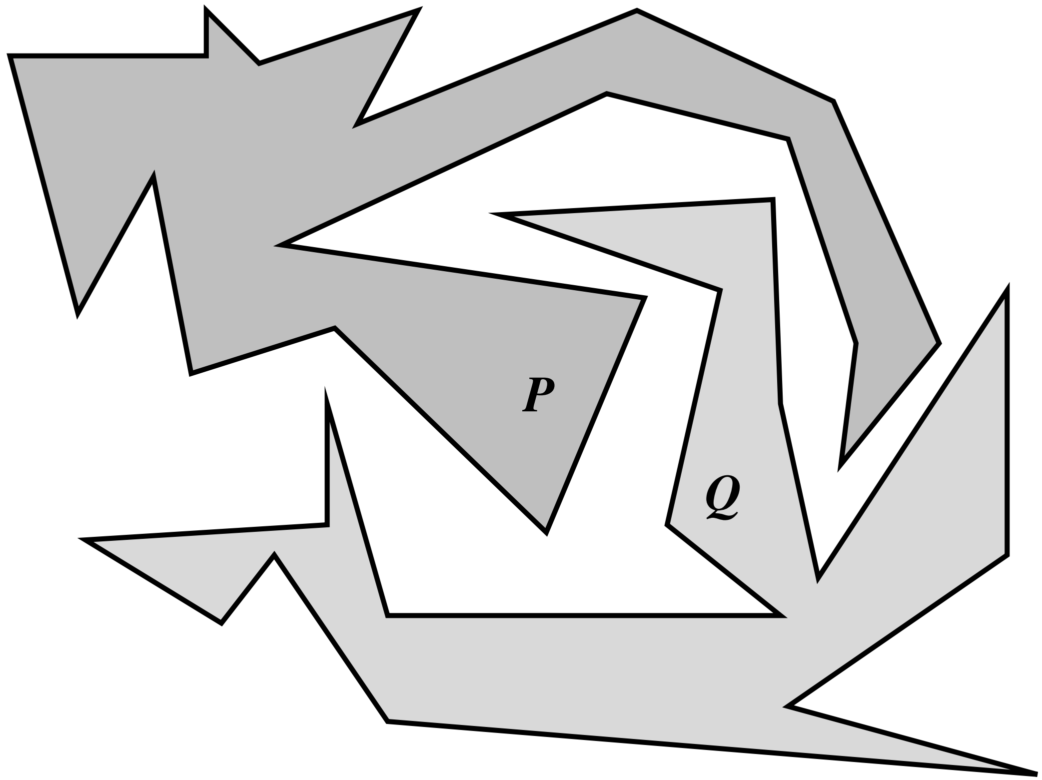



Euclidean TSP

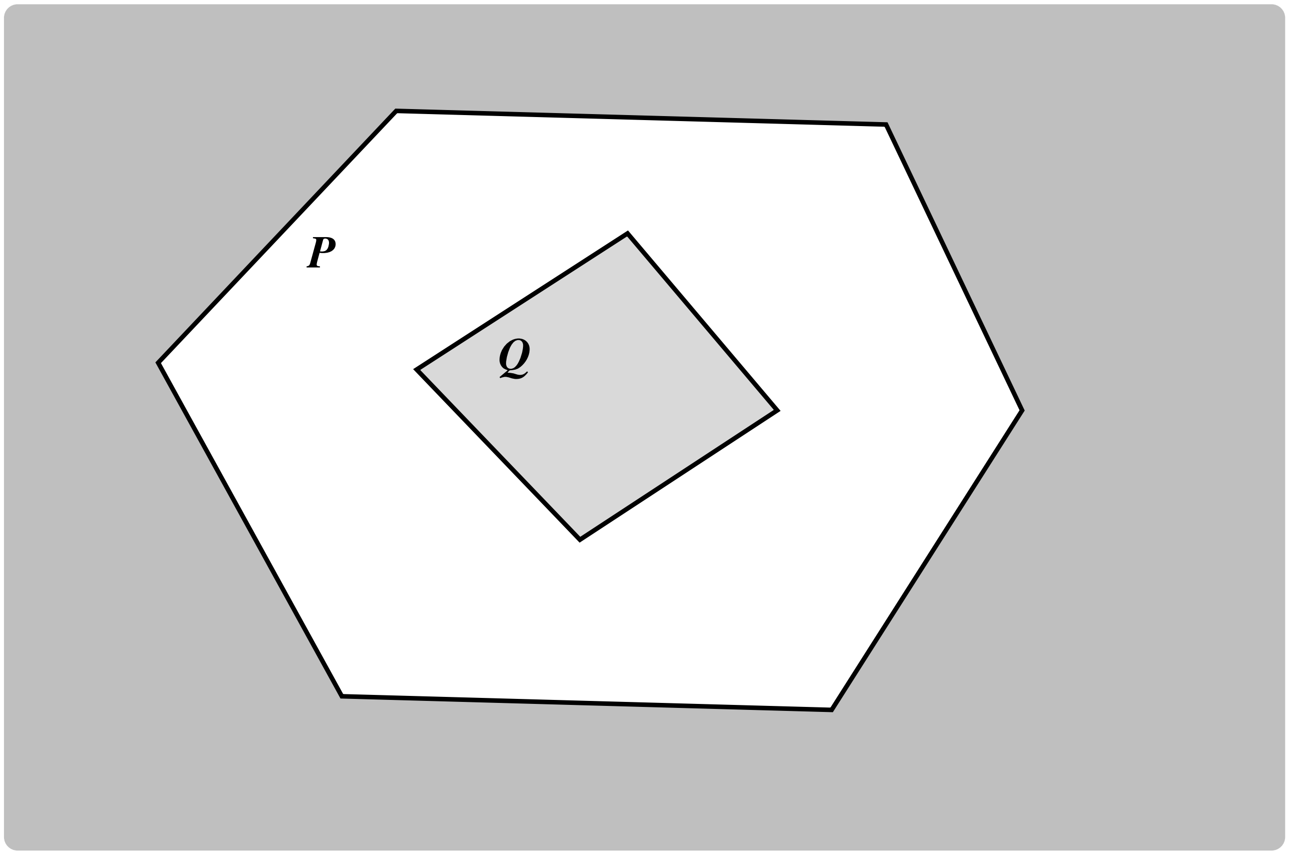
# Exact Solutions

- Constant TSP
- Distance metric constraints
- Polygon and line/lines

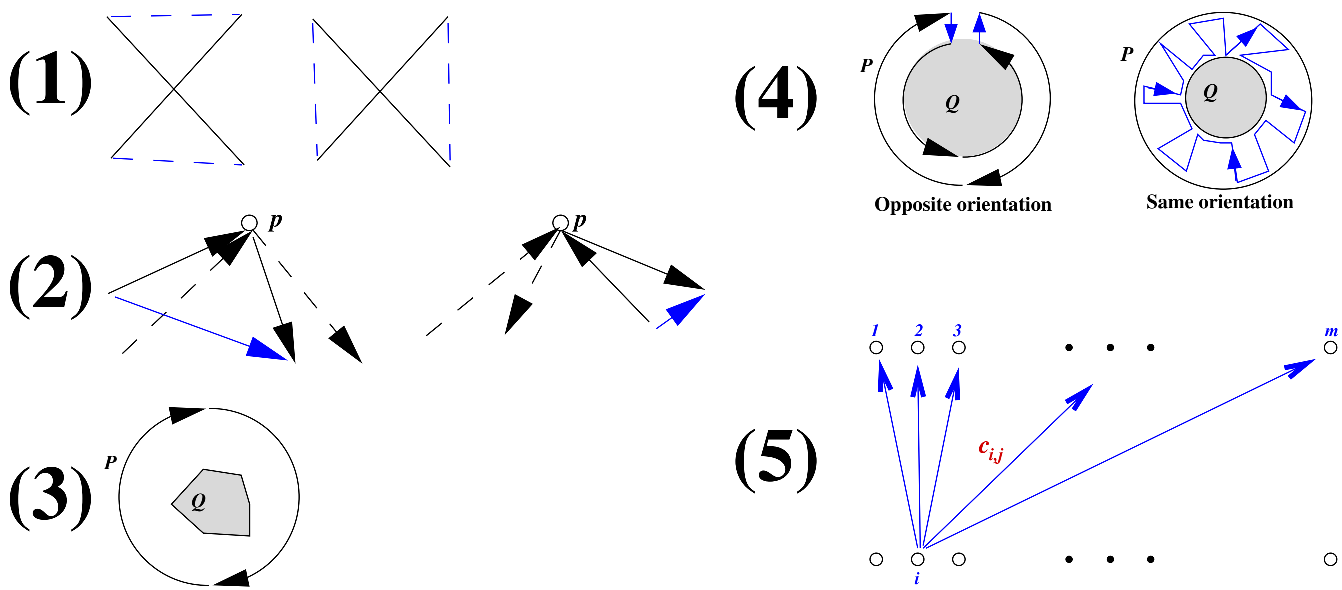
# Our Case: Two Polygons



# Our Simple Case: Convex

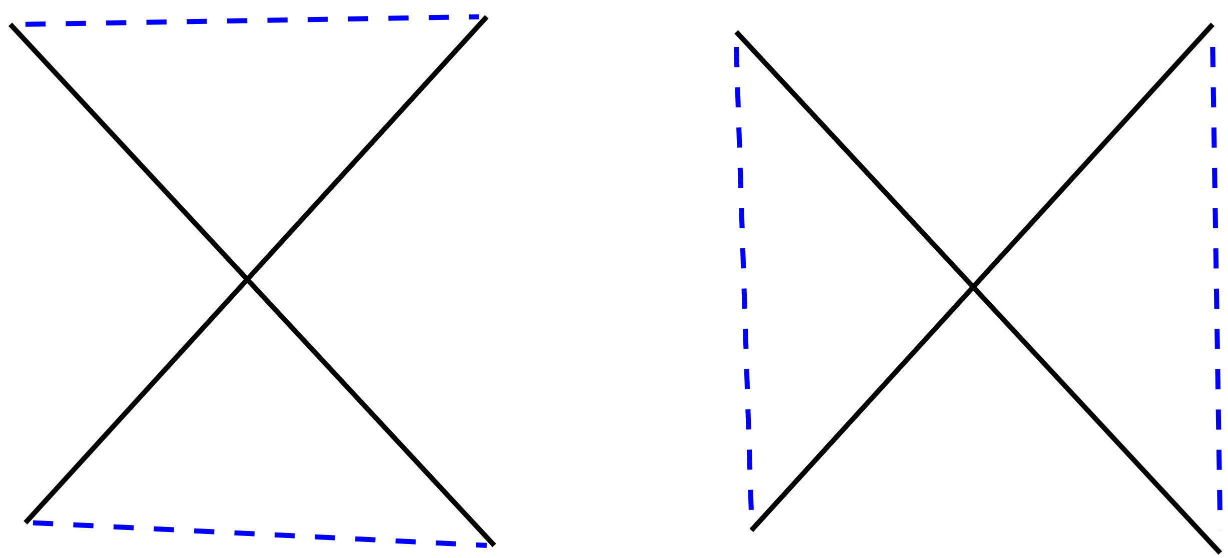


# Shortest Tours (Convex): Proof Sketch



# Shortest Tours (Convex): Intersections

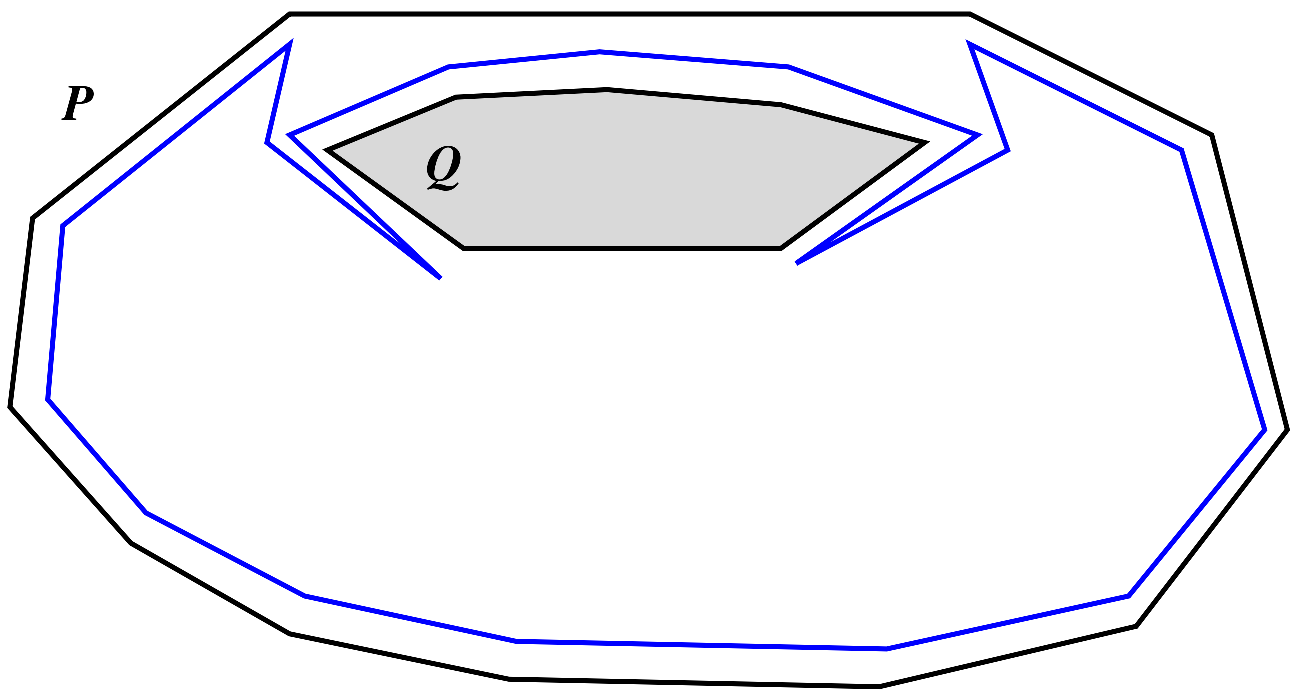
Can't cross itself like this:



# Shortest Tours (Convex): Intersections

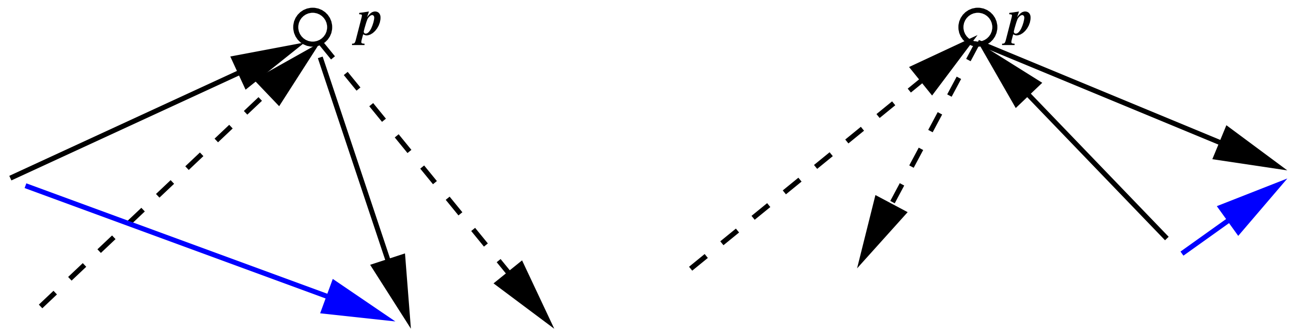
- No intersections on $P$ vertices
- Intersections on $Q$ vertices

# Shortest Tours (Convex): Intersections



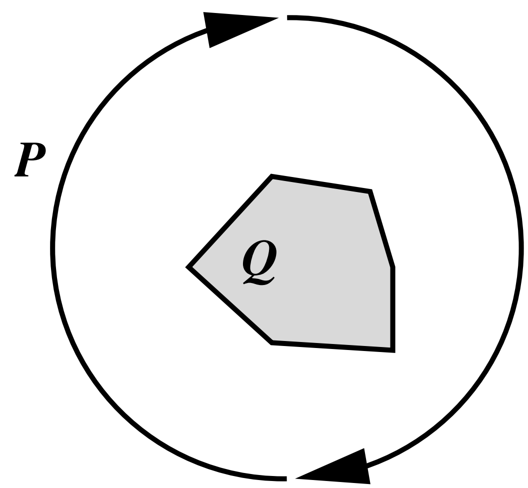
# Shortest Tours (Convex): Intersections

Some shortest tour visits each $P$ vertex precisely once.



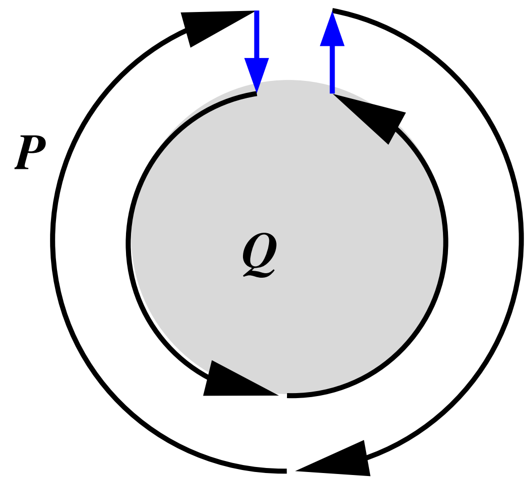
# Shortest Tours (Convex): Orientation

Some shortest tour goes around $P$ in cyclic order



# Shortest Tours (Convex): Orientation

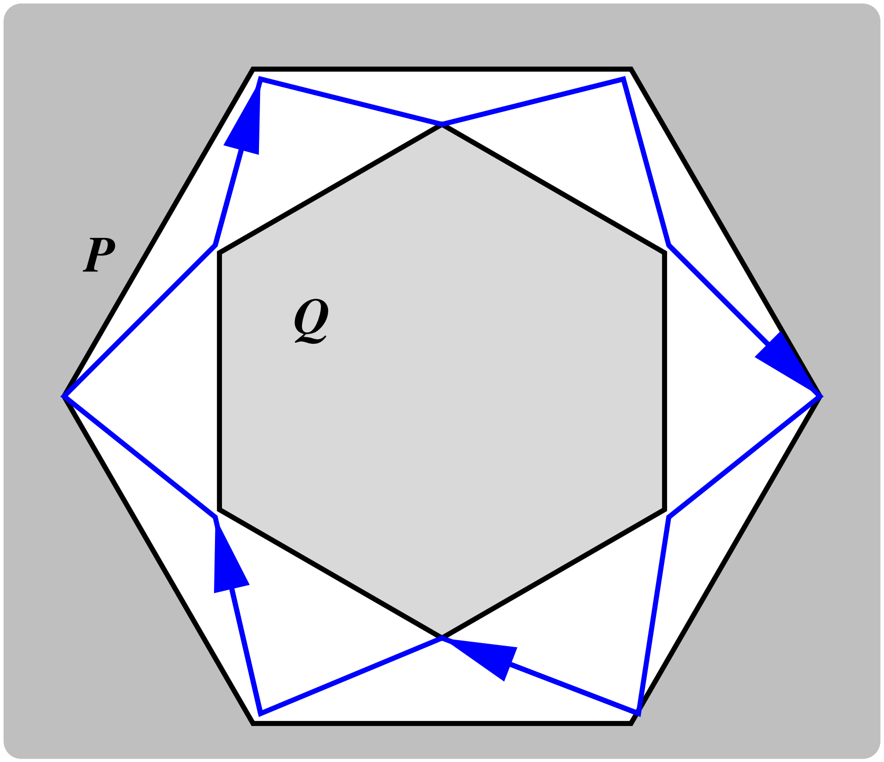
If a shortest tour is clockwise on $P$ and counter-clockwise on $Q$, then there is precisely one detour.



# Shortest Tours (Convex): Detours

- Think of a shortest tour as a set of detours.
- Might need $\min(|P|, |Q|)$ detours

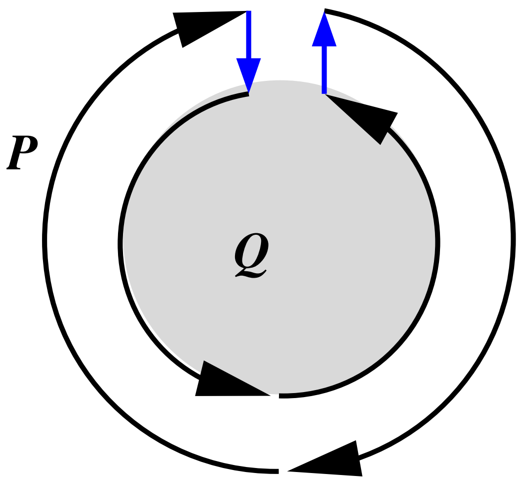
# Shortest Tours (Convex): Detours



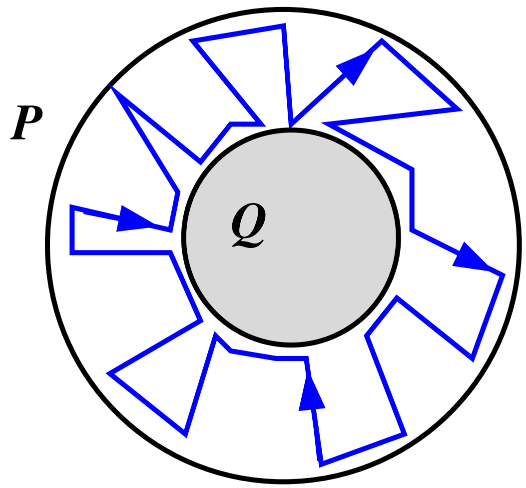

Might need $\min(|P|, |Q|)$ detours.

# Shortest Tours (Convex): Detours

What we know:




**Opposite orientation**

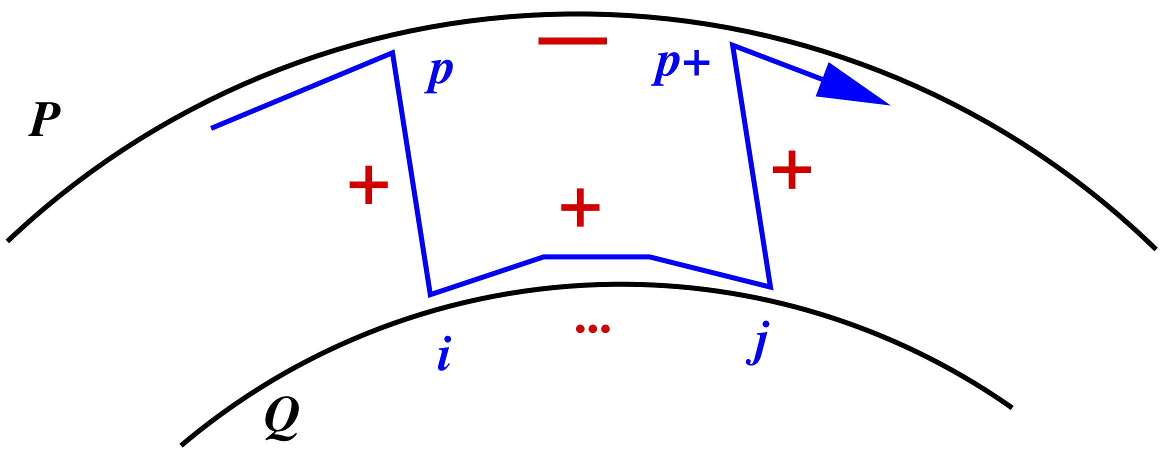



**Same orientation**

# Shortest Tours (Convex): Detours

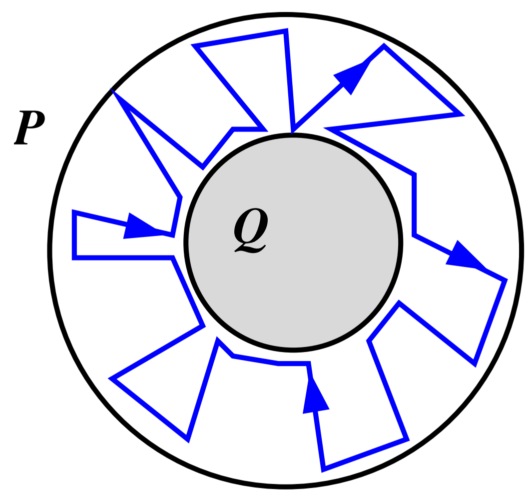
Call $d^p_{i,j}$ the detour from $p$ through $i \leadsto j$. Let $c^p_{i,j}$ be its cost.

Let $c_{i,j}$ and $d_{i,j}$ be minima (over $p \in P$).

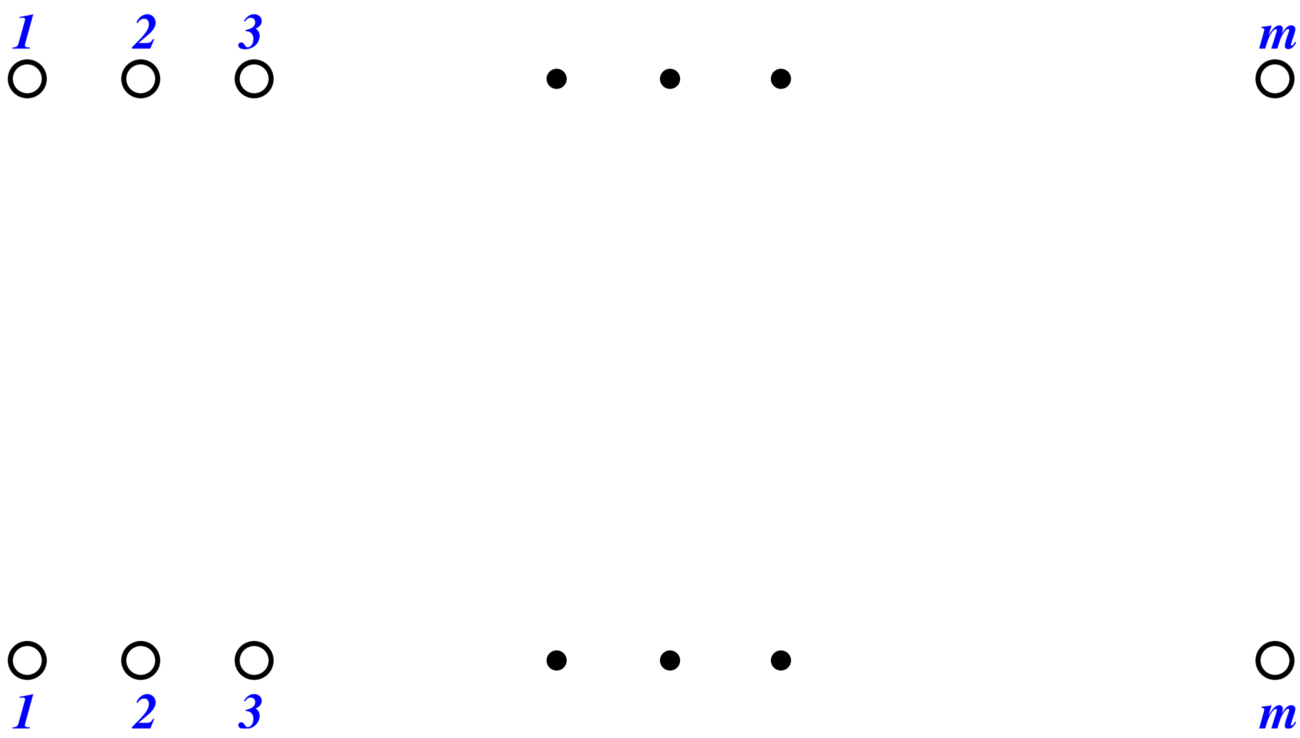


# Shortest Tours (Convex): Detours

A same-orientation shortest tour goes from one detour to the next.



# Shortest Tours (Convex): Shortest Paths



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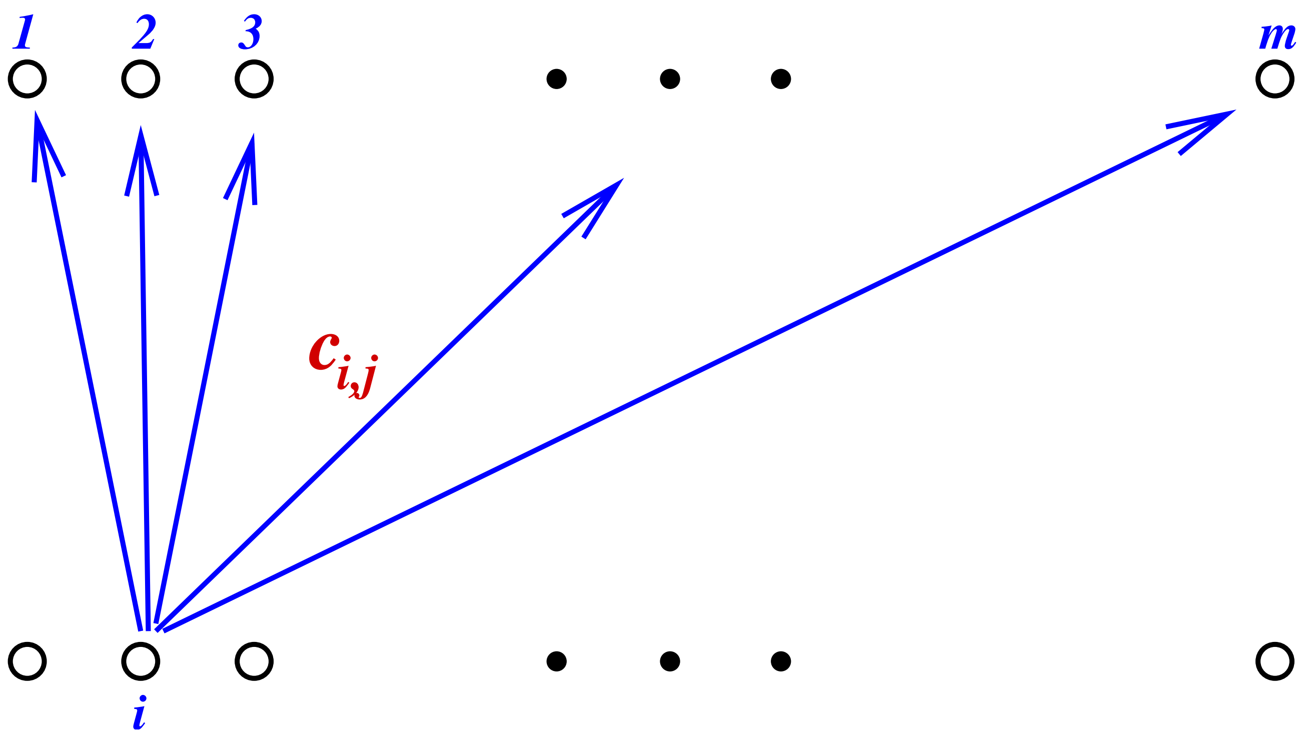
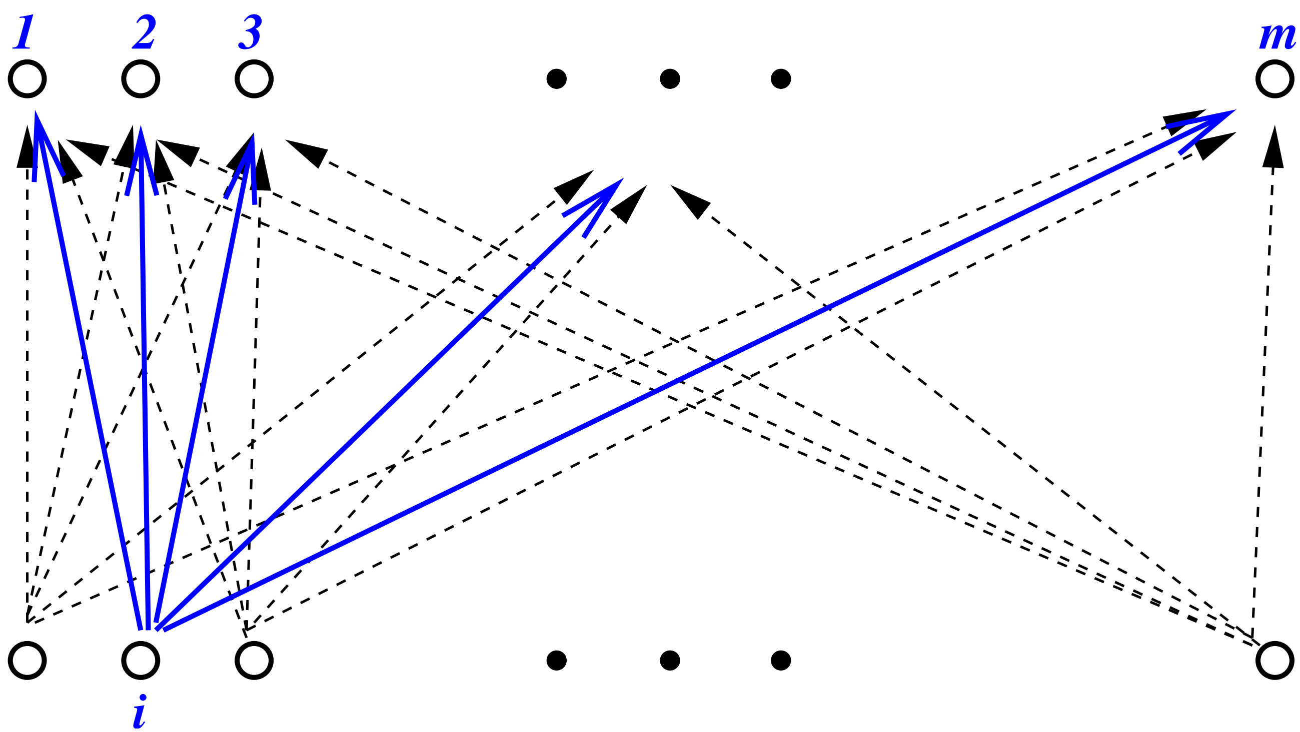
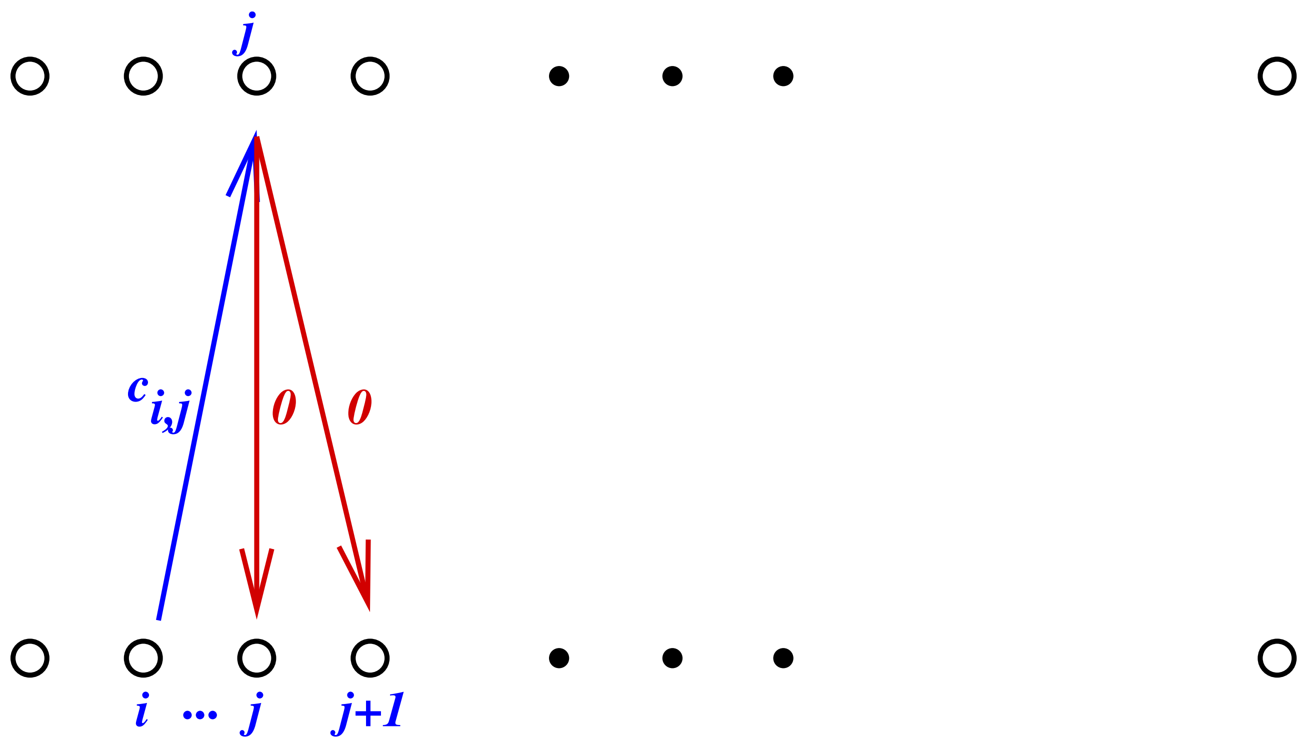


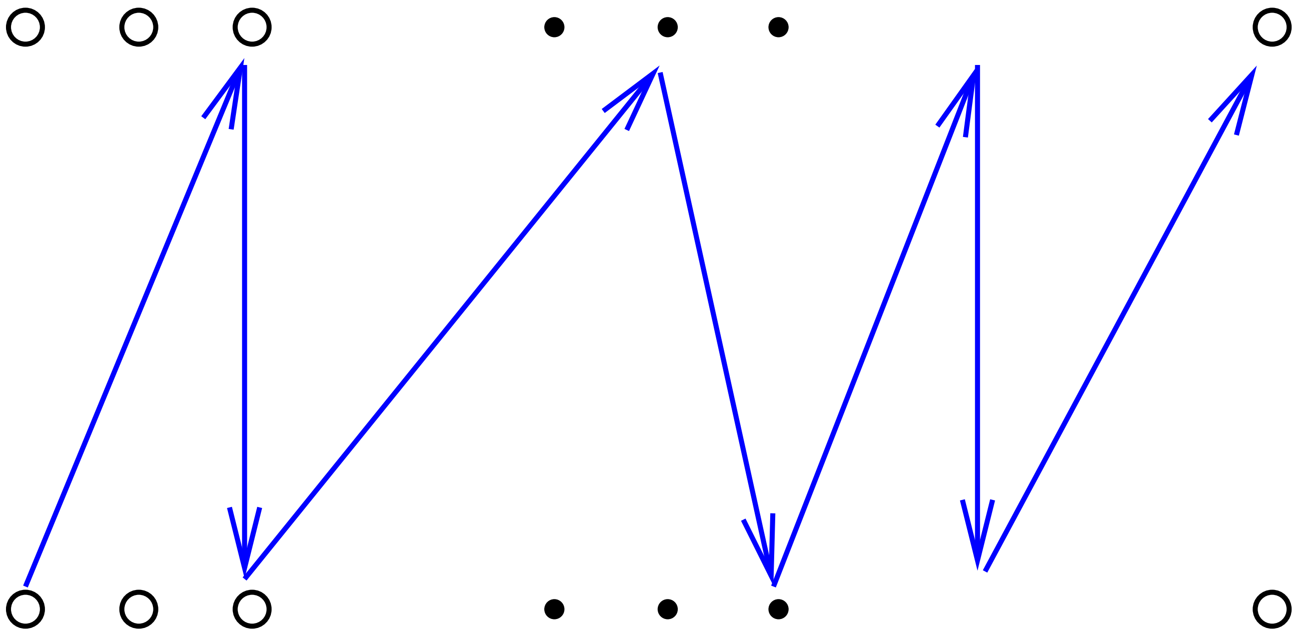
# Shortest Tours (Convex): Shortest Paths



# Shortest Tours (Convex): Shortest Paths



# Shortest Tours (Convex): Shortest Paths



# Shortest Tours (Convex): Shortest Paths

Find $m$ shortest paths (vertices mod $m$):

- $1 \rightsquigarrow m$
- $2 \rightsquigarrow 1$
- $3 \rightsquigarrow 2$
- etc.

# Shortest Tours (Convex): Done!

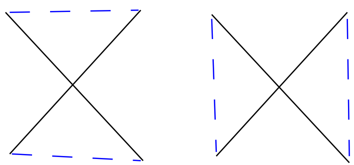
Summary so far:

- $m$ candidate same orientation tours
- 1 candidate opposite orientation tour

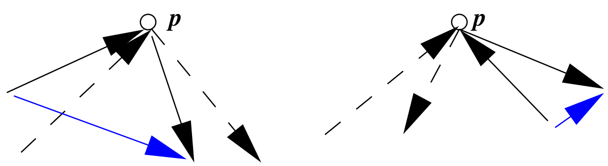
Pick the minimum over $m + 1$ tours!

# Shortest Tours (Convex): Recap

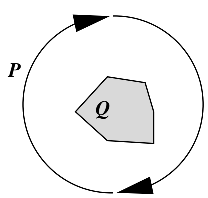
**(1)**



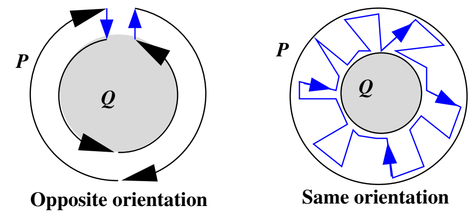
**(2)**



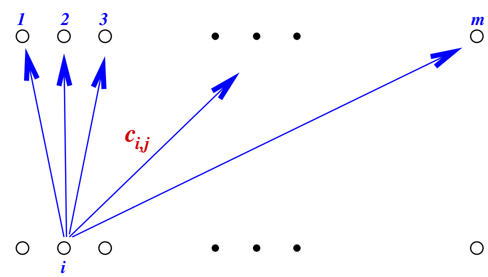

**(3)**




**(4)**



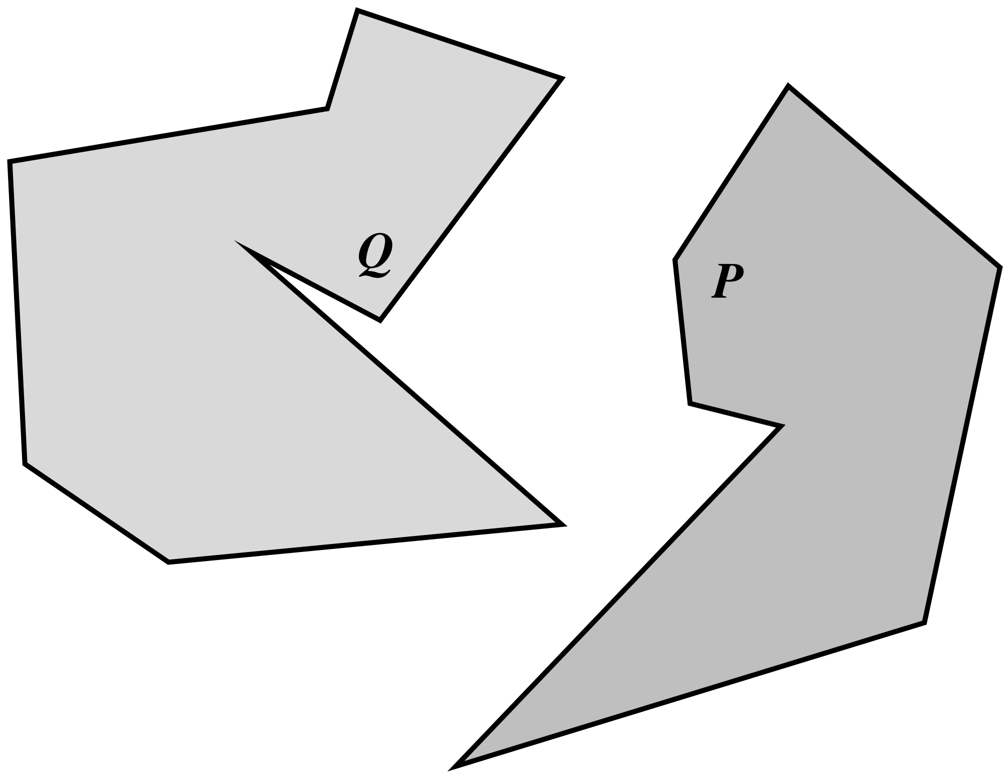

**(5)**



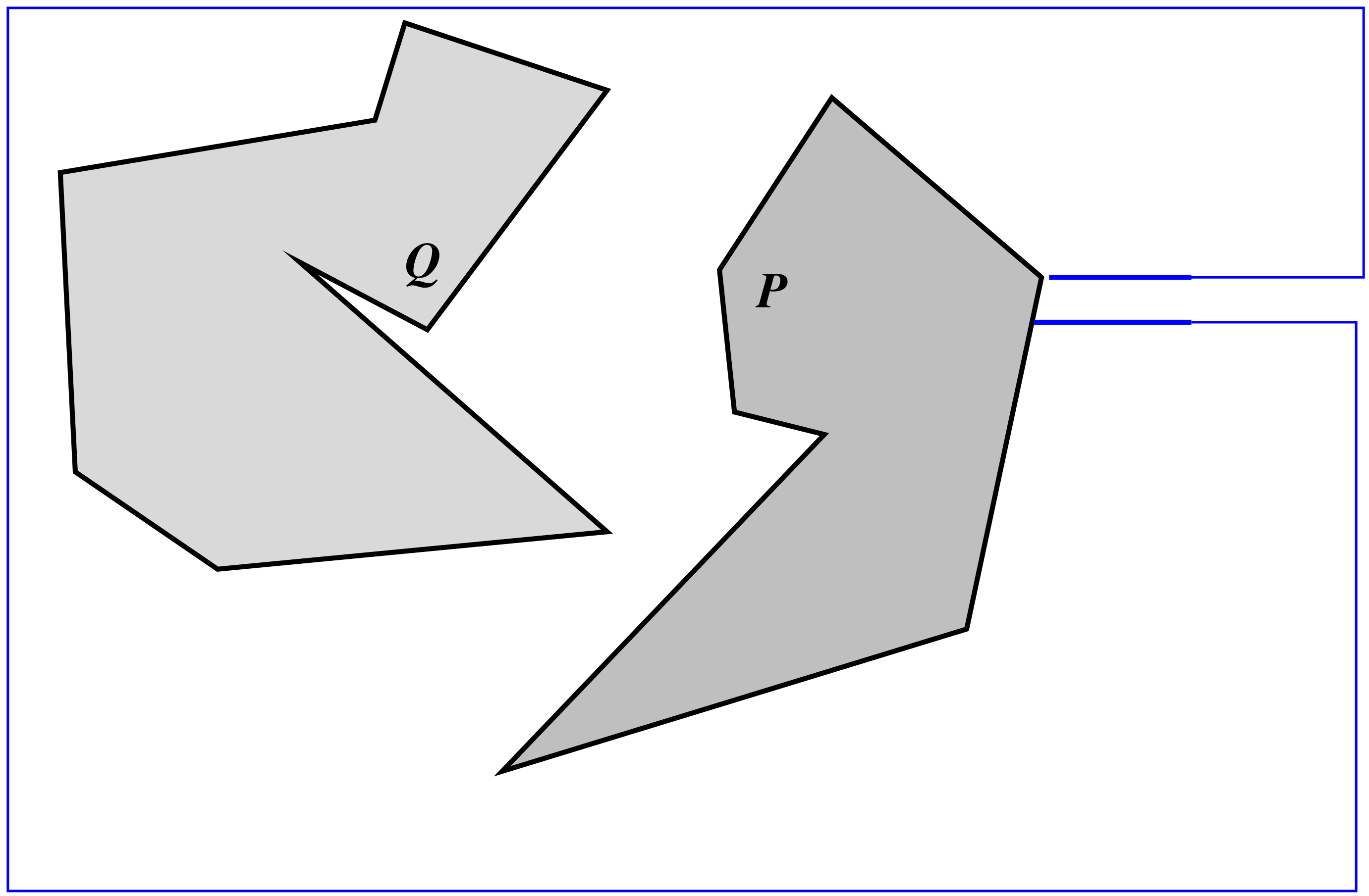
# Shortest Tours: Non-Convex Nested

- Same ideas
- Visibility more complicated
- Intersections on $P$ vertices as well as $Q$
- Backtracking on $P$ as well as on $Q$
- Cyclic on $P$ other than backtracking

# Non-Convex, Non-Nested Polygons



# Non-Convex, Non-Nested Polygons

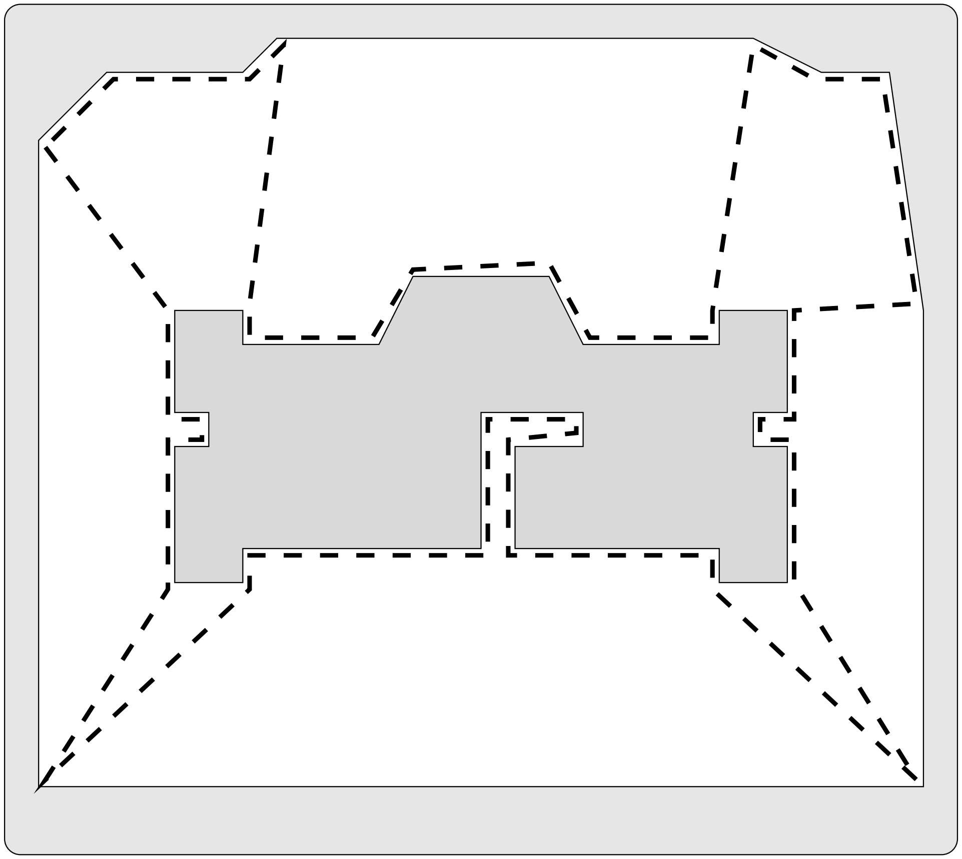


# Robots

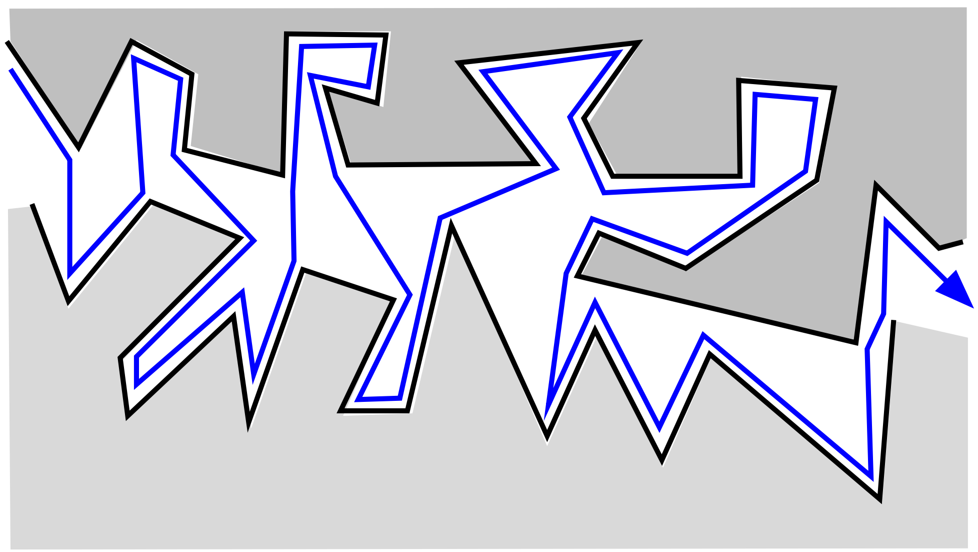
- Cool
- High Tech
- Good demos

Navigation.

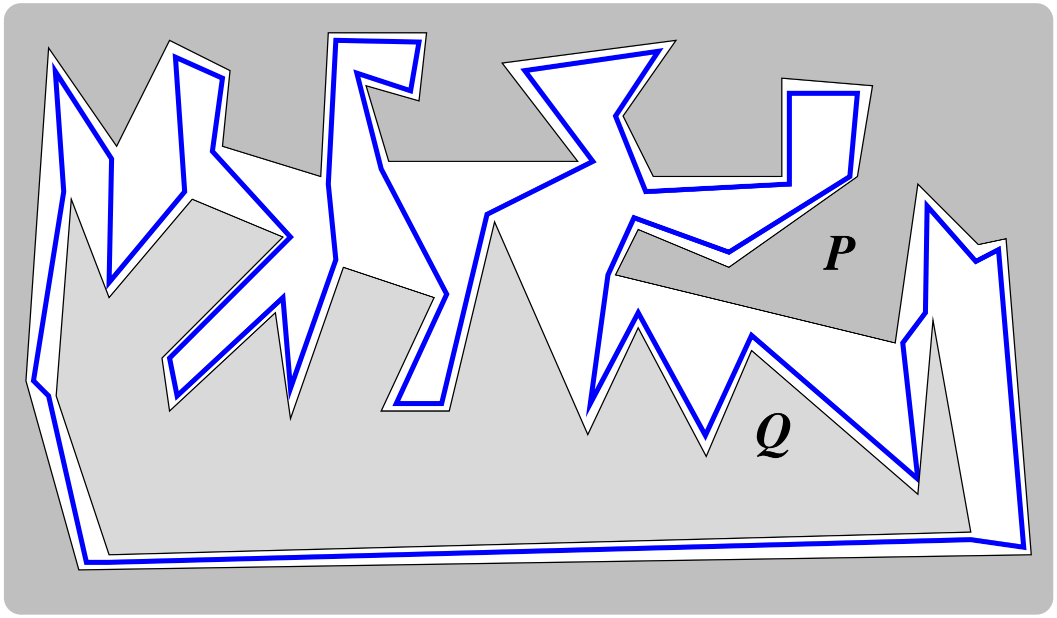
# Robots: Building Perimeters



# Robots: Channel Navigation



# Robots: Channel Navigation



# More?

$k > 2$ polygons?

- General ETSP in the limit
- Not clear how to generalize detour technique
- Convex hulls might intersect, so can't simplify in general

# Acknowledgments

Work with Ali Shokoufandeh (Drexel) and Pawel Winter (Copenhagen)

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# Questions