

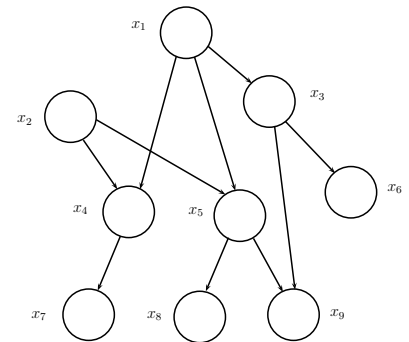
Homework #2

Lecturer: Prof. Ko Nishino

Counts for 15% of final grade

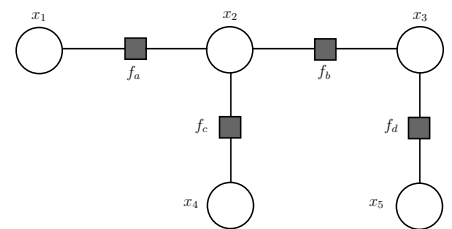
You must solve the following problems solely by yourself. You are expected to support all your answers with clear and detailed arguments, derivations, sketches and/or proofs. Please clearly number your solution with the corresponding problem number and staple all the solution sheets together. No late submission will be accepted without prior permission by the instructor.

Problem 1 Write down the joint probability distribution of the graphical model shown on the right. Show whether the following conditional independence actually hold:



1. when x_7 is observed, $x_2 \perp\!\!\!\perp x_9 | x_3$,
2. when x_1 is observed, $x_6 \perp\!\!\!\perp x_7 | x_5$.

Problem 2 Write down the unnormalized joint distribution of the factor graph shown on the right. Then, derive the marginal distribution of x_3 using the sum-product algorithm.



Problem 3 Consider a tree-structured factor graph, in which a given subset of the variable nodes form a connected subgraph (i.e., any variable node of the subset is connected to at least one of the other variable nodes via a single factor node). Show how the sum-product algorithm can be used to compute the marginal distribution over that subset.

Problem 4 Consider a special case of a Gaussian mixture model in which the covariance matrices Σ_k of the mixture components are all constrained to have a common value $\hat{\Sigma}$. Derive the EM equations for maximizing the likelihood function under such a model.

Problem 5 Consider a density model given by a mixture distribution

$$p(\mathbf{x}) = \sum_{k=1}^K \pi_k p(\mathbf{x}|k)$$

and suppose that we partition the vector x into two parts so that $x = (x_a, x_b)$. Show that the conditional density $p(x_b|x_a)$ is itself a mixture distribution and find expressions for the mixing coefficients and for the component densities.