

λ -Calculus Problems

1. $(\lambda x.a (x y))$
normal form

2. $((\lambda x. * x x)5)$

$$\begin{aligned} &\xrightarrow{\beta} (* 5 5) \\ &\xrightarrow{\beta} 25 \end{aligned}$$

3. $a (\lambda a.a b) a$
normal form

4. $(\lambda x. \lambda y. + (x 3) (y 5)) (\lambda x. * x 3)$
 $\xrightarrow{\beta} (\lambda y. + ((\lambda x. * x 3) 3) (y 5))$
 $\xrightarrow{\beta} (\lambda y. + ((* 3 3)) (y 5))$
 $\xrightarrow{\beta} (\lambda y. + (9) (y 5))$

5. $(\lambda y. (\lambda x. + x y) y) 5$
 $\xrightarrow{\beta} (\lambda x. + x 5) 5$
 $\xrightarrow{\beta} (+ 5 5)$
 $\xrightarrow{\beta} 10$

6. $(\lambda x.x x x) (\lambda x.x x x)$
 $\xrightarrow{\beta} (\lambda x.x x x) (\lambda x.x x x) (\lambda x.x x x)$
...

7. $(\lambda x.x x) a (\lambda x.x x)$
 $\xRightarrow{\beta} a a (\lambda x.x x)$
8. $(\lambda x.((\lambda y. (* 2 y))(+ x y))) y$
 $\xRightarrow{\beta} ((\lambda y. (* 2 y))(+ y y))$
 $\xRightarrow{\beta} (* 2 (+ y y))$
 $\xRightarrow{\beta} (* 2 (2 y))$
 $\xRightarrow{\beta} (* 4 y)$
9. $(\lambda x. \lambda y. \lambda z. x y z)(\lambda x. \lambda y. x) a b$
 $\xRightarrow{\beta} (\lambda y. \lambda z. (\lambda x. \lambda y. x) y z) a b$
 $\xRightarrow{\beta} (\lambda z. (\lambda x. \lambda y. x) a z) b$
 $\xRightarrow{\beta} (\lambda y. a) b$
 $\xRightarrow{\beta} a$
10. $(\lambda x. a ((\lambda y.x y) (a x))) (a y)$
 $\xRightarrow{\alpha} (\lambda x.a ((\lambda z.x z) (a x))) (a y)$
 $\xRightarrow{\beta} (a ((\lambda z.(a y) z) (a (a y))))$
 $\xRightarrow{\beta} a (a y) (a (a y))$