MATH 642, Spring 2000

Problem Set 1

Due Tuesday, May 2

1. Show that the conditions on the window $g(t-n)$ can be written as

$$\begin{array}{cc} g^2(t)+g^2(-t)=1 & -\frac{1}{2}\le x\le\frac{1}{2} \\ g(t)=g(1-t) & \frac{1}{2}\le x\le\frac{3}{2} \\ g(t)=0 & \text{elsewhere.} \end{array}$$

   This yields $g(t)=1$ in $[a,1-a]$ where there is no overlap.

2. Draw a linear function $f(t)$ on $[0,1]$ and its folding $h(t)$.

3. Show that the two-dimensional upsampling with quincunx matrix $\mathbf{M}_q=\begin{bmatrix}1 & 1\\ -1 & 1\end{bmatrix}$,

$$\mathbf{u}(\mathbf{n})=\mathbf{x}(\mathbf{M}_q^{-1}\mathbf{n})$$

   yields

$$U(\omega_1,\omega_2)=\frac{1}{2}(X(\omega_1,\omega_2)+X(\omega_1+\pi,\omega_2+\pi)).$$

   Which inputs give $\mathbf{u}=\mathbf{0}$?