Introduction to Computer Vision

Week 8, Fall 2010
Instructor: Prof. Ko Nishino
Midterm

![Bar chart showing points distribution for midterm students]
Project 2

without radial distortion correction

with radial distortion correction
Light Light Light

- How do you recover the true color of a scene?
  - Recovering Lightness

- How do you describe reflection?
  - Radiometry and Reflectance
Recovering Lightness
Land’s Experiment (1959)

- Cover all patches except a blue rectangle
- Make it look gray by changing illumination
- Uncover the other patches

**Color Constancy**

We filter out illumination variations
Color Cube Illusion

D. Purves, R. Beau Lotto, S. Nundy “Why We See What We do,” American Scientist
Checker Shadow Illusion

by Ted Adelson @ MIT
Lightness Recovery and Retinex Theory

- Recover surface reflectance/color in varying illumination conditions
  - Use tools developed before
    - Sensing: intensity/color
    - Image processing: Fourier transform and convolution
    - Edge operators
    - Iterative techniques
Assume monochromatic light $\lambda = \lambda_i$ and $q(\lambda_i) = 1$

$$b'(x,y) = r'(x,y)e'(x,y)$$

Reflectance as a multiplier to irradiance is often called lightness

Note: our analysis can be applied to colored light (we’ll simply have filters in front of the sensor)
Also, assume everything is flat
Lightness Recovery

\[ b'(x,y) = r'(x,y)e'(x,y) \]

Can we recover \( r' \) and \( e' \) from \( b' \)?

We need assumptions to constrain the solution!

- Sharp changes in reflectance \( r'(x,y) \)
- Smooth changes in illumination \( e'(x,y) \)

Frequency spectrum (Fourier transform)

We want to filter out \( e' \)
Lightness Recovery

\[ b'(x, y) = r'(x, y)e'(x, y) \]

Take logarithm

\[ \log b'(x, y) = \log r'(x, y) + \log e'(x, y) \]

\[ b(x, y) = r(x, y) + e(x, y) \]

Use Laplacian

\[ d = \nabla^2 b = \nabla^2 r + \nabla^2 e \]
\[ \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \]

- Sharp changes in reflectance \( r'(x, y) \)
  \( \nabla^2 r \) has 2 infinite spikes near edges and \( \nabla^2 r = 0 \) elsewhere

- Smooth changes in illumination \( e'(x, y) \)
  \( \nabla^2 e \approx 0 \) everywhere
Lightness Recovery (Retinex Scheme)

Image:
\[ b = r + e \]

Laplacian:
\[ d = \nabla^2 b = \nabla^2 r + \nabla^2 e \]

Thresholding:
\[ t = T(d) \approx \nabla^2 r \]

Reconstruction:
\[ l(x, y) = k + r(x, y) \]  
(lightness) (reflectance)
### Solving the Inverse Problem

<table>
<thead>
<tr>
<th>Image</th>
<th>Laplacian</th>
<th>Thresholding</th>
<th>Lightness</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b = r + e$</td>
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<td>$l(x, y)$</td>
</tr>
</tbody>
</table>

Find lightness $l(x, y)$ from $t(x, y)$

Poisson’s Equation

$$\nabla^2 l = t$$

$$\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) l(x, y) = t(x, y)$$

We have to find $g(x, y)$ which satisfies

$$l(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} t(u, v) g(x - u, y - v) \, du \, dv$$

$$l(x, y) = t(x, y) * g(x, y)$$
Solving Poisson’s Equation

We have

\[ \nabla^2 l(x,y) = t(x,y) \quad \text{and} \quad l(x,y) = t(x,y) * g(x,y) \]

Fourier transform

\[ ? = T(u,v) \quad \text{and} \quad L(u,v) = T(u,v)G(u,v) \]

So

\[ -(u^2 + v^2)L(u,v) = T(u,v) \quad \text{and} \quad L(u,v) = T(u,v)G(u,v) \]

Thus

\[ G(u,v) = -\frac{1}{u^2 + v^2} \quad \rightarrow \quad g(x,y) = \frac{1}{2\pi} \log \sqrt{x^2 + y^2} + c \]

See Robot Vision page 123,124
Lightness Recovery

\[ g(x,y) = \frac{1}{2\pi} \log \sqrt{x^2 + y^2} \]

\[ l(x,y) = k + r(x,y) \]

(lightness)  (reflectance)

Which means: \[ l'(x,y) = kr'(x,y) \]

Normalize:

Assume maximum value of \( l'(x,y) = l'_\text{max} \) corresponds to \( r' = 1 \)

Then:

\[ r'(x,y) = \frac{l'(x,y)}{l'_\text{max}} \quad (\text{reflectance}) \]

\[ e'(x,y) = \frac{b'(x,y)}{r'(x,y)} \quad (\text{illumination}) \]
Computing Lightness (Discrete Case)

Log of image

\( b(x, y) \rightarrow T(d) \rightarrow \nabla^2 b \)

\[ \begin{bmatrix} 1 & 4 & 1 \\ 4 & -20 & 4 \\ 1 & 4 & 1 \end{bmatrix} \]

basically inverse of Laplacian mask

But, inverse of a discrete approximation of the Laplacian would not be an exact inverse of the Laplacian

\[ g(x, y) = \frac{1}{2\pi} \log \sqrt{x^2 + y^2} \]

Solve \( \nabla^2 l = t \) directly
Computing Lightness (Discrete Case)

\[ \frac{1}{6} \begin{bmatrix} 1 & 4 & 1 \\ 4 & -20 & 4 \\ 1 & 4 & 1 \end{bmatrix} \rightarrow \nabla^2 l = t \rightarrow T \left( \frac{1}{6} \begin{bmatrix} 1 & 4 & 1 \\ 4 & -20 & 4 \\ 1 & 4 & 1 \end{bmatrix} \ast b \right) \]

\[-20l_{i,j} + 4 \left( l_{i+1,j} + l_{i,j-1} + l_{i-1,j} + l_{i,j-1} \right) + \left( l_{i+1,j+1} + l_{i-1,j+1} + l_{i-1,j-1} + l_{i+1,j-1} \right) = 6 t_{i,j} \]

Solve iteratively

\[ l^{n+1}_{i,j} = \frac{1}{5} \left( l^n_{i+1,j} + l^n_{i,j+1} + l^n_{i-1,j} + l^n_{i,j-1} \right) + \frac{1}{20} \left( l^n_{i+1,j+1} + l^n_{i-1,j+1} + l^n_{i-1,j-1} + l^n_{i+1,j-1} \right) - \frac{3}{10} t_{i,j} \]

Use a discrete approximation to the inverse of Laplacian to obtain initial estimate of \( l \)
Lightness from Multiple Images taken under Varying Illumination

Illumination is not smooth

Use spatial statistics of edges

Derivative operator responses are sparse

\[ \nabla b(x,y,t) = \nabla r(x,y) + \nabla e(x,y,t) \]

\[ \nabla r(x,y) \approx \text{median}_{t} (\nabla b(x,y,t)) \]

Y. Weiss ICCV 2001
Lightness from Multiple Images taken under Varying Illumination

Y. Weiss ICCV 2001
Lightness from Multiple Images taken under Varying Illumination

frame 1
frame 11
ML reflectance
ML illumination 1
ML illumination 2

Y. Weiss ICCV 2001
Using Lightness

Y. Weiss ICCV 2001
Using Lightness

Tracking result (1)

Original image sequence
Preprocessed image sequence using our method

Y. Matsushita and K. Nishino CVPR 2003
Comments on Lightness Recovery

- Not applicable to smooth reflectance variations
- Not applicable to curved objects

In general

\[ \text{Intensity} = f(\text{Shape, Reflectance, Illumination}) \]
Appearance

- How do you describe reflection?
  - Radiometry and Reflectance

- Can we recover geometry from shading? When can we do that?
  - Photometric Stereo
Radiometry and Reflectance
Radiometry and Image Formation

To interpret image intensities, we need to understand Radiometric Concepts and Reflectance Properties

- Image Intensities: Overview
- Radiometric Concepts:
  - Radiant Intensity
  - Irradiance
  - Radiance
  - Bidirectional Reflectance Distribution Function (BRDF)
- Image Formation

Radiometry vs. Photometry?
Light!

- **Geometric Optics**
  - Light rays
  - Emission, Reflection/Refraction, Absorption

- **Wave Optics**
  - Electromagnetic wave (Maxwell’s equations)
  - Diffraction, Interference, Polarization

- **Quantum Optics**
  - Photons
  - Fluorescence, Phosphorescence
Geometric Optics

Emission  Reflection  Refraction  Absorption
Snell’s law

\[ n_1 \sin \theta_1 = n_2 \sin \theta_2, \]

\[ \frac{n_2}{n_1} = \frac{\sin \theta_1}{\sin \theta_2} \]

\( n_1 = 1 \) for air
Image Intensities

Image Intensity = \( f(\text{orientation } \mathbf{n}, \text{surface reflectance, illumination}) \)

Image intensity understanding is an under-constrained problem!
Radiometric Concepts

Solid Angle

$\omega : \text{Solid Angle subtended by } dA$

$$d\omega = \frac{dA'}{R^2} = \frac{dA \cos \theta_i}{R^2} \quad \text{(str)}$$

$dA'$ : Foreshortened Area

What is the solid angle subtended by a hemisphere?
Radiometric Concepts

Radiant Intensity
Light flux (power) emitted per unit solid angle

\[ J = \frac{d\phi}{d\omega} \quad (\text{W/str}) \]

\( d\phi \): Flux

Surface Irradiance
Light flux (power) incident per unit surface area

\[ E = \frac{d\phi}{dA} \quad (\text{W/m}^2) \]

Does not depend on where the light is coming from
Radiometric Concepts

Surface Radiance (Brightness)
Light flux (power) emitted per unit foreshortened area per unit solid angle

\[ L = \frac{d\phi}{(dA \cos \theta_r) d\omega} \quad \text{(W/m}^2\text{str)} \quad d\phi : \text{Flux} \]

- \( L \) depends on direction \( \theta_r : L(\theta_r) \)
- Surface can radiate into whole hemisphere
- \( L \) is proportional to irradiance \( E \)
- Depends on reflectance properties of surface
Sun Example

Total flux: $3.91 \times 10^{26} \text{W}$

Surface area: $6.07 \times 10^{18} \text{m}^2$

Radiance

$$L = \frac{d\phi}{(dA \cos \theta_r) d\omega} \quad \left( \text{W/m}^2\text{str} \right)$$

Total Flux

$$\int d\phi = L \int dA \int \cos \theta d\omega$$

$$L = \frac{3.91 \times 10^{26}}{\int dA \int \cos \theta r d\omega}$$

by Steve Marschner
Differential Solid Angles

\[ dA = (r \, d\theta)(r \sin \theta \, d\phi) = r^2 \sin \theta \, d\theta \, d\phi \]

\[ d\omega = \frac{dA}{r^2} = \sin \theta \, d\theta \, d\phi \]

\[ S = \int_0^\pi \int_0^{2\pi} \sin \theta \, d\theta \, d\phi = 4\pi \]
Sun Example

Total flux: $3.91 \times 10^{26}$ W

Surface area: $6.07 \times 10^{18}$ m$^2$

Radiance

$$L = \frac{d\phi}{(dA \cos \theta_r) d\omega} \left( \text{W/m}^2\text{str} \right)$$

Total Flux

$$\int d\phi = L \int dA \int \cos \theta d\omega$$

$$L = \frac{3.91 \times 10^{26}}{6.07 \times 10^{18} \times \pi} = 2.05 \times 10^{7} \left( \text{W/m}^2\text{str} \right)$$

by Steve Marschner
Sun Example

- Same flux on Earth and Mars?
Sun Example

- Same flux on Earth and Mars?

\[ d\phi = LdA\cos\theta d\omega \]

Radiance does not change along the ray!

\[ \frac{1}{r^2} \] fall off of flux
Radiometric Image Formation
(Scene Radiance to Image Irradiance)

Image Irradiance: \( E \)
Scene Radiance: \( L \)

Solid angles: \( d\omega_i = d\omega_s \)

\[
\frac{dA_i \cos \alpha}{(f / \cos \alpha)^2} = \frac{dA_s \cos \theta}{(z / \cos \alpha)^2}
\]

\[
\therefore \frac{dA_s}{dA_i} = \frac{\cos \alpha}{\cos \theta} \left( \frac{z}{f} \right)^2
\]
Radiometric Image Formation
(Scene Radiance to Image Irradiance)

Image Irradiance: \( E \)  
Scene Radiance: \( L \)

Solid Angle subtended by lens

\[
d\omega_L = \frac{\pi d^2 \cos \alpha}{4 \left(\frac{z}{\cos \alpha}\right)^2}
\]
Radiometric Image Formation
(Scene Radiance to Image Irradiance)

Image Irradiance: \( E \)
Scene Radiance: \( L \)
Flux received by lens from \( dA_s = \) Flux projected onto \( dA_i \) (image)

\[
d\phi = L(dA_s \cos \theta) d\omega_L
\]

Image Irradiance
\[
E = \frac{d\phi}{dA_i}
\]
Radiometric Image Formation
(Scene Radiance to Image Irradiance)

We have

Solid Angles:
\[
\frac{dA_s}{dA_i} = \frac{\cos \alpha}{\cos \theta} \left( \frac{z}{f} \right)^2
\]

Solid Angle subtended by lens:
\[
d\omega_L = \frac{\pi d^2}{4} \cos \alpha \left( \frac{z}{\cos \alpha} \right)^2
\]

Flux:
\[
d\phi = L(dA_s \cos \theta)d\omega_L
\]

Image Irradiance:
\[
E = \frac{d\phi}{dA_i}
\]

We get
\[
E = L \frac{\pi}{4} \left( \frac{d}{f} \right)^2 \cos^4 \alpha
\]

- Image irradiance is proportional to scene radiance
- Telephoto lenses have narrow field of view
  → effects of $\cos^4 \alpha$ are small