1. **(Finite Abelian Groups)** Show how to get all abelian groups of order $2^33^45^2$ up to isomorphism. (Hint: use fundamental theorem of finite abelian group)

2. **(Finite Abelian Groups)** Prove that every finite abelian group can be expressed as a direct product of cyclic groups of order $n_1, n_2, \ldots, n_t$, where $n_{i+1}$ divides $n_i$ for $i = 1, 2, \ldots, t - 1$. (Hint: prove that if $m$ and $n$ are relatively prime, then $C(m) \times C(n)$ is isomorphic to $Cmn$)

3. **(Definition of Ring)** Let $R$ be a ring. If every $x \in R$ satisfies $x^2 = x$, prove that $R$ must be commutative. (Hint: consider the expansion of $(a \pm b)^2$ and show that for all $x \in R$ we have $x + x = 0$)

4. **(Ring and Group isomorphisms)** Let $2\mathbb{Z}$ and $3\mathbb{Z}$ be set of all multiples of 2 and the set of all multiples of 3 in $\mathbb{Z}$ respectively, where $\mathbb{Z}$ is the ring of integers. Answer and justify the following:
   (a) Is $2\mathbb{Z}$ isomorphic to $3\mathbb{Z}$ as additive abelian groups?
   (b) Is $2\mathbb{Z}$ isomorphic to $3\mathbb{Z}$ as rings?

5. **(Chinese Remainder Theorem)** We want to solve a system of congruence equations. For example: let us ask the question: Does the system $\begin{cases} x = 5 \pmod{60} \\ x = 4 \pmod{11} \end{cases}$ have a solution for $x$ in $\mathbb{Z}$? To answer such a question, prove that if $m$ and $n$ are relatively prime integers and $a$ and $b$ are any integers, there exists an integer $x$ such that $x = a \pmod{mn}$ and $x = b \pmod{n}$. (Hint: consider the remainders of $a, a + m, a + 2m, \ldots, a + (n-1)m$ on division by $n$).

6. **(Ideal)** Let $R$ be a commutative ring with a unit element. Recall the definition of prime ideal as we discussed in the lecture. Prove that
   (a) Every maximal ideal of $R$ is a prime ideal.
   (b) The converse of (a) is not true in general. However, if we assume that $R$ is a finite set, then it is true. Prove it.