The questions given here are taken from old exams. Their number does not necessarily reflect the length of your exam on January 29, but it isn't far off. Not included here are questions on determinants and linear transformations.

1. Find a matrix $A$ such that
   
   $A\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} x - y \\ x + y + 2w \end{bmatrix}$

2. (a) Find a LU-decomposition of the matrix using the elementary matrix approach outlined in class. Verify that $A = LU$ after you find the LU decomposition.
   
   $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & 4 \end{bmatrix}$

   (b) Solve $Ax = b$ using only the LU decomposition when
   
   $b = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

3. Calculate $B$ when $(AB)^{-1}A^2 = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/3 \end{bmatrix}$ and $A = \begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix}$. Hint: Use rules of matrix algebra and simplify first.

4. (a). Find the row reduced form of the matrix $A$
   
   $A = \begin{bmatrix} 1 & -1 & 1 & 0 \\ 1 & 1 & -1 & 2 \end{bmatrix}$

   (b) Use the results of part (a) and give the **vector parametric solution** of $Ax = 0$.

5. Let $C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ and $A = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$.

   (a) Give a simplified form of $A^TCA$.

   (b) Solve the linear system $A^TCAx = -2$ for $x$.

6. (a) Is the vector $b$ in the span of the columns of $A$? Give verifiable reasons.
   
   $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ 3 \\ 3 \end{bmatrix}$

   (b) Answer the same question if $b = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$