1. (25 points) Integrate \( \int_C \text{Re}(2z + i)dz \), where \( C \) is the parabolic path along the parabola \( y = x^2 \) from \(-1 + i\) to \(1 + i\).

**Solution:** A parametrization for the path is: \( z(t) = t + it^2, -1 \leq t \leq 1 \). Inserting this into the integrand we have \( \text{Re}(2z + i) = \text{Re}(2(t + it^2) + i) = \text{Re}(2t + i2t^2 + i) = 2t \) and \( dz = (1 + 2it)dt \).

Hence the integral
\[
\int_C \text{Re}(2z + i)dz = \int_{-1}^{1} (2t)(1 + 2it)dt = \int_{-1}^{1} (2t + 4it^3) \Big|_{-1}^{1} = (1 + 4i) - (1 + 4i) = \frac{8i}{3}.
\]

2. (25 points) Evaluate using Stokes’ Theorem \( \oint_C zdx + xdy + ydz \), where \( C \) is the trace of the cylinder \( x^2 + z^2 = 3 \) in the plane \( y = 3 \).

**Solution:** We write the line integral in vector form \( \oint_C \mathbf{F} \cdot d\mathbf{r} \), where \( \mathbf{F} = zi + xj + xyk \). By Stokes’ theorem we have \( \oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_S \text{curl} \mathbf{F} \cdot \mathbf{n} dA \), where \( S \) is the planar surface in \( y = 3 \) bounded by \( C \). Now \( \text{curl} \mathbf{F} = \begin{vmatrix} i & j & k \\ \partial_x & \partial_y & \partial_z \\ z & x & xy \end{vmatrix} = xi - j(y - 1) + k \), and obviously the unit surface normal for \( S \) is \( j \), i.e., \( \mathbf{n} = j \). Inserting these in the integral and carrying out the dot product we have
\[
\iint_S \text{curl} \mathbf{F} \cdot \mathbf{n} dA = \iint_S -(y - 1) dA = \iint_S -(3 - 1) dA = -2 \iint_S dA
= -2 (\text{area of } S) = -2(\pi(\sqrt{3})^2) = -6\pi.
\]

3. (25 points) Find the one/ones that is analytic and express its derivative in terms of the variable \( z \).

*Justify your answer.* (Hint: Use the Cauchy-Riemann equations or else)

(a) \( f(z) = |e^z| \), \( z = x + iy \)  
(b) \( g(z) = \text{Re}(z^2 + z) + i\text{Im}(z^2 + z) \), \( z = x + iy \)

**Solution:** (a) \( f(z) = |e^z| = e^u \) so that \( u = e^x \), and \( v = 0 \) \( \Rightarrow u_x = e^x, v_y = 0 \) so the Cauchy-Riemann equations are violated. We conclude that \( f(z) \) is nonanalytic. (b) By definition \( g(z) = z^2 + z \), so \( g(z) \) is analytic. And \( g'(z) = 2z + 1 \).

4. (25 points) Find (a) \( \text{Im}(i\ln(2 - i)) \), where \( \ln \) denotes the principal value of the complex logarithm.

(b) \( \text{Re}(i\sin(2 + 3i)) \)

**Solution:** (a) We compute \( i\ln(2 - i) \) first. By definition
\[
i\ln(2 - i) = i(\ln|2 - i| + i\text{Arg}(2 - i)) = i(\ln \sqrt{5} + i\text{Arg}(2 - i)) = -\text{Arg}(2 - i) + i\ln \sqrt{5} = \text{Im}(i\ln(2 - i))
\]

(b) Using addition formula we have
\[
\sin(2 + 3i) = (\sin 2)(\cosh 3) + i(\cos 2)(\sinh 3) \Rightarrow i\sin(2 + 3i) = -(\cos 2)(\sinh 3) + i(\sin 2)(\cosh 3).
\]

So \( \text{Re}(i\sin(2 + 3i)) = -\cos(2)(\sinh 3) \).