1. (25 points) Calculate the line integral \( \int_C xy \, dx + x^2y^2 \, dy \), where \( C \) is the portion of the parabola \( y = 2x^2 \) from \((0,0)\) to \((1,2)\).

**Solution:** We use the parametrization for the curve \( C \):
\[
\begin{align*}
x &= t \\
y &= 2t^2
\end{align*}
\]
with \( 0 \leq t \leq 1 \). Hence \( dx = dt \) and \( dy = 4tdt \).

Plugging these expressions in the line integral we get
\[
\int_C xy \, dx + x^2y^2 \, dy = \int_0^1 (2t^3) \, dt + t^2(2t^2)^2(4tdt).
\]
After simplification we obtain
\[
\left. \int_0^1 (2t^3 + 16t^7) \, dt = \left( \frac{1}{4}t^4 + 2t^8 \right) \right|_0^1 = \frac{5}{2}.
\]

2. (25 points) The line integral in question is path independent. Evaluate the integral by using its potential function.

\[
\int_{(1,2)}^{(2,3)} 2y^2 \, dx + 4xy \, dy
\]

**Solution:** By definition the potential function \( f(x,y) \) satisfies the following system of equations:
\[
\begin{align*}
(a) \quad \frac{\partial f}{\partial x} &= 2y^2 \\
(b) \quad \frac{\partial f}{\partial y} &= 4xy
\end{align*}
\]
After integrating equation \( (a) \) with respect to \( x \) we get
\[
f = \int 2y^2 \, dx = 2y^2x + C(y).
\]
To determine the function \( C(y) \) we differentiate the above equation with respect to \( y \) and use the equation \( (b) \). Thus
\[
4xy + C'(y) = 4xy \Rightarrow C'(y) = 0 \Rightarrow C(y) = K, \text{ a constant.}
\]
Hence the potential function \( f = 2xy^2 + K \). Then we invoke the fundamental theorem of line integral to get
\[
\int_{(1,2)}^{(2,3)} 2y^2 \, dx + 4xy \, dy = \int_{(1,2)}^{(2,3)} \nabla f \cdot dr = f(-2,3) - f(1,2) = -36 - 8 = -44.
\]