1. (25 points)
Use Green’s theorem to evaluate the line integral \( \oint_C F \cdot dr \), where the vector field \( F = yi + xyj \) and the contour \( C \) is the triangular path with vertices \((0,0),(1,2),(1,0)\). You must use Green’s theorem to do the problem. No other method will be accepted.

**Solution:** \[ \oint_C F \cdot dr = \int_C ydx + xydy = \iint_R (y-1)dA = \int_0^1 \int_{y/2}^{y-1} (y-1)dydx, \text{ or } \int_0^1 \int_{2x} (y-1)dx. \]

The value of the integral is \( \frac{2}{3} \).

2. (25 points) This problem deals with the interchanging the order of integration.

The moment of inertia \( I_x \) with respect to the \( x \)-axis of the upper semidisc \( R : x^2 + y^2 \leq 1, y \geq 0 \) is given as the double integral below

\[ \iint_R y^2dA \]

Convert the above integral into iterated integrals:

\[ \int_0^1 \int_{\sqrt{1-x^2}}^{\sqrt{1-y^2}} y^2dydx = \iint_R y^2dA = \int_0^1 \int_{\sqrt{1-x^2}}^{1} y^2dxdy \]

Note that in the above equation there are eight question marks which need to be determined.

Determine the expressions in the question marks in the limits of integration. No evaluation of the integral is required for the problem.

**Solution:** \[ \iint_R y^2dA = \int_0^1 \int_{\sqrt{1-x^2}}^{\sqrt{1-y^2}} y^2dxdy = \int_0^1 \int_{\sqrt{1-x^2}}^{1} y^2dxdy. \]