1. Determine the value for each of the following integrals. Each integral has equal weight.

(a) \[ \oint_{C_1} \frac{z+1}{z^2(z-2)} \, dz \]

where contour \( C_1 \) is depicted below.

**Solution:** The shapes of the contours are referred to what were given on the test. We don’t draw them here. The singularities of the integrand are 0, and 2 and they are all enclosed in \( C_1 \). By principle of deformation of path we can replace \( C_1 \) by contours \( C_3 \) and \( C_0 \), where \( C_3 \) encloses 0 and \( C_0 \) encloses 2 respectively. Now by Cauchy integral formula we have

\[ \oint_{C_1} \frac{z+1}{z^2(z-2)} \, dz = \oint_{C_3} \frac{(z+1)/((z-2)z)}{z^2} \, dz = 2\pi i \frac{d}{dz} \left[ \frac{z+1}{z-2} \right] \bigg|_{z=0} = 2\pi i \left[ \frac{-3}{(z-2)^2} \right] \bigg|_{z=0} = -\frac{3\pi i}{2}. \]

Similarly

\[ \oint_{C_0} \frac{z+1}{z^2(z-2)} \, dz = \oint_{C_0} \frac{(z+1)/(z^2)}{(z-2)} \, dz = 2\pi i \frac{d}{dz} \left[ \frac{z+1}{z-2} \right] \bigg|_{z=2} = \frac{3\pi i}{2}. \]

Consequently \( \oint_{C_1} \frac{z+1}{z^2(z-2)} \, dz = \oint_{C_3} \frac{z+1}{z^2(z-2)} \, dz + \oint_{C_0} \frac{z+1}{z^2(z-2)} \, dz = \left( -\frac{3\pi i}{2} \right) + \left( \frac{3\pi i}{2} \right) = 0. \)

(b) \[ \oint_{C_2} \frac{z+1}{z^2(z-2)} \, dz \]

where contour \( C_2 \) is depicted below.

**Solution:** The contour \( C_2 \) encloses no singularities at all, by Cauchy integral theorem

\[ \oint_{C_2} \frac{z+1}{z^2(z-2)} \, dz = 0. \]

(c) \[ \oint_{C_3} \frac{z+1}{z^2(z-2)} \, dz \]

where contour \( C_3 \) is depicted below.

**Solution:** This integral is exactly the same as \( \oint_{C_3} \frac{z+1}{z^2(z-2)} \) as was given in the solution of part (a). Hence its value is \( -\frac{3\pi i}{2} \).