1. This problem is to verify Cauchy integral theorem.
   (a) Consider the contour integral \( \oint_C \frac{e^z}{z^2 + 9} \, dz \) where the contour \( C \) : Start from 0 going along the parabola \( y = x^2 \) to reach the point \( 1 + i \), then return to the origin by moving along the parabola \( y = \sqrt{x} \).

   To do direct evaluation for the integral \( \int_C \frac{e^z}{z^2 + 9} \, dz \) we need to break the contour \( C \) into two parts \( C_1 \) and \( C_2 \), where \( C_1 \) moves from 0 to \( 1 + i \) along the parabola \( y = x^2 \), and \( C_2 \) comes back to 0 along the other parabola. That is, \( \int_C \frac{e^z}{z^2 + 9} \, dz = \int_{C_1} \frac{e^z}{z^2 + 9} \, dz + \int_{C_2} \frac{e^z}{z^2 + 9} \, dz \). To evaluate \( \int_{C_1} \frac{e^z}{z^2 + 9} \, dz \) we parametrize \( C_1 : z(t) = t + it^2, 0 \leq t \leq 1 \). This gives \( dz = (1 + 2it) \, dt \). And insert this parametrization into the integral for evaluation. In this way \( \int_{C_1} \frac{e^z}{z^2 + 9} \, dz \) is converted into an ordinary integral with complex integrand \( \int_0^1 \frac{e^{t+it^2}}{(t+it^2)^2 + 9} \, (1 + 2it) \, dt \). We now call for Maple for evaluating \( \int_0^1 \frac{e^{t+it^2}}{(t+it^2)^2 + 9} \, (1 + 2it) \, dt \). This is how \( \int_{C_1} \frac{e^z}{z^2 + 9} \, dz \) is computed directly. What is the value of \( \int_{C_1} \frac{e^z}{z^2 + 9} \, dz \)?

   (b) In a similar way we can compute \( \int_{C_2} \frac{e^z}{z^2 + 9} \, dz \). What is the value of \( \int_{C_2} \frac{e^z}{z^2 + 9} \, dz \)?

   (c) The original contour integral \( \oint_C \frac{e^z}{z^2 + 9} \, dz \) is the sum of them. What is the value of \( \oint_C \frac{e^z}{z^2 + 9} \, dz \)? Does the result of your calculation for \( \oint_C \frac{e^z}{z^2 + 9} \, dz \) agree with what Cauchy integral theorem predicts?

2. This problem is to verify Cauchy integral formula.
   We now change the contour \( C \) in the integral in problem 1 to the circle \( |z| = 5 \) and still retain the same integrand \( \frac{e^z}{z^2 + 9} \). That is, we want to compute the contour integral \( \oint_{|z|=5} \frac{e^z}{z^2 + 9} \, dz \). An obvious parametrization for the contour is \( z(t) = 5e^{it}, 0 \leq t \leq 2\pi \). You may want to use this to carry out a direct numerical evaluation using Maple. The procedure is just the same as in problem 1.

   (a) What is the result of your numerical calculation for the integral \( \oint_{|z|=5} \frac{e^z}{z^2 + 9} \, dz \)?

   (b) What is the theoretical value of the integral as implied by Cauchy integral formula? Your numerical computation need to confirm this theoretical value.