1. (25 points) Evaluate the line integral \( \int_C y \, dx + x^2 y \, dy \), where \( C \) is the portion of the parabola \( y^2 = 2x \) from \((0,0)\) to \((2,2)\). You must show me the conversion of the line integral into an ordinary integral.

**Solution:** A parametrization of the curve can be chosen as
\[
x = \frac{t^2}{2}, \quad y = t
\]
for \(0 \leq t \leq 2\).

So, the integral
\[
\int_C y \, dx + x^2 y \, dy = \int_0^2 f(t) \, dt + \frac{t^5}{4} \, dt
\]
\[
= \left[ \frac{t^3}{3} + \frac{t^6}{24} \right]_0^2 = \frac{16}{3}
\]

2. (25 points) Show that the line integral in question is path-independent. Evaluate the integral by using its potential function.

\[
\int_{(1,0)}^{(-2,1)} 4xe^y \, dx + 2x^2 e^y \, dy
\]

**Solution:** The integral can be written as
\[
\int_{(1,0)}^{(-2,1)} 4xe^y \, dx + 2x^2 e^y \, dy = \int P \, dx + Q \, dy
\]

where \( P = 4xe^y, \quad Q = 2x^2 e^y \).

We compute \( \frac{\partial P}{\partial y} = 4xe^y \) and \( \frac{\partial Q}{\partial x} = 4xe^y \). Thus we see that
\[
\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}
\]

Hence the integral is path-independent.

To evaluate the integral a potential function \( f(x,y) \) can found by inspection: i.e.
\[
f(x,y) = 2x^2 e^y
\]

This function \( f \) obvious satisfies
\[
\nabla f = Pi + Qj
\]

Thus qualified as a potential function for \( Pi + Qj \).

By fundamental theorem we get
\[
\int_{(1,0)}^{(-2,1)} 4xe^y \, dx + 2x^2 e^y \, dy = \int_{(1,0)}^{(-2,1)} \nabla f \cdot dr = f(-2,1) - f(1,0)
\]
\[
= 2(-2)^2 e - 2(1)^2 e^0 = 8e - 2 \approx 19.746
\]