1. (25 points) Find the number of integral solutions to the following equation so that 
\[ x_1 \geq 1, x_2 \geq 2, x_3 \geq 3, \text{ and } x_4 \geq 4 \]
\[ x_1 + x_2 + x_3 + x_4 = 1000 \]

Solution: The equation can be equivalently written as
\[ (x_1 - 1) + (x_2 - 2) + (x_3 - 3) + (x_4 - 4) = 900 \]
Let 
\[ x'_1 := x_1 - 1, x'_2 := x_2 - 2, x'_3 := x_3 - 3, x'_4 := x_4 - 4 \]
So equation (1) is turned to
\[ x'_1 + x'_2 + x'_3 + x'_4 = 900 \]
where the constraints is the ordinary \( x'_i \geq 0 \), for \( 1 \leq i \leq 4 \). The procedure can obviously be reversed. This means that the number of solutions to (2) is equal to the number of solutions to (1).
The number of solutions for equation (2) is \( \frac{993!}{(990)!(3)!} \). Hence the answer is \( \frac{993!}{(990)!(3)!} \).

2. (25 points) Use Binomial theorem to determine the coefficient of the \( x \) term in the expansion of
\[ (3x - \frac{2}{x})^{99} \]

Solution: By binomial theorem we have
\[ (3x - \frac{2}{x})^{99} = \sum_{k=0}^{99} C(99, k)(3x)^k\left(-\frac{2}{x}\right)^{99-k} \]
\[ = \sum_{k=0}^{99} C(99, k)3^k(-2)^{99-k}x^{-99+2k} \]
Note that the \( k \)th term in the sum is \( C(99, k)3^k(-2)^{99-k}x^{-99+2k} \). For the coefficient of \( x \) term we need to choose \( k \) such that \( x^{-99+2k} = x \). This implies
\[ -99 + 2k = 1 \Rightarrow k = 50 \]
Hence the coefficient of \( x \) term is
\[ C(99, 50)3^{50}(-2)^{49} \]
\[ = \frac{99!}{50!49!} 3^{50}(-2)^{49} \]