Ordinary function defining parametric motion on a line.

Graph version of the same function produces a curve in the plane.
Parametric version of function with 1 input and two outputs.

\[ f(t) = (x(t), y(t)). \]
Examples of Curves defined parametrically.

1. \( x = \cos(t), \ y = \sin(t), \ 0 \leq t \leq \frac{\pi}{2} \)

The parametric equations define not only a curve, but a method of moving along the curve.
2. \( x = \cos(t), \ y = \sin(t), \ 0 \leq t \leq 2\pi \)
3. \( x = \cos(t), \ y = \sin(t), \ -\infty \leq t \leq \infty \)
Here $y=x^2$ so we get part of a parabola.
5. \( x = \sin(t), y = \sin(t)^2, \quad 0 \leq t \leq \infty \)

Here we start at 0, and repeatedly move back and forth on the parabola \( y = x^2 \)
Here we start at negative infinity, and move over the entire parabola to infinity.
7. \( x = t - 3\sin(t), y = 4 - 3\cos(t), 0 \leq t \leq 10 \)
8. \[ x = \cos t + (1/2)\cos 7t + (1/3)\sin 17t \]
\[ y = \sin t + (1/2)\sin 7t + (1/3)\cos 17t \]
\[ (=5 \leq t \leq 5) \]
The cycloid: Find the equation of the curve traveled by a point on the circumference of a wheel when the wheel rolls.

\[ x = a\theta - a\sin\theta, \quad y = a - a\cos(\theta), 0 \leq \theta \leq 2\pi \]
The brachistochrone problem:

Of all possible paths from A to B (above), which one has the property that a bead sliding down such a wire without friction will reach B in the shortest time? (Quickest ride down part of a roller coaster)
The tautochrone problem:

Of all possible paths from A to B (above), which one has the property that a bead sliding down such a wire without friction will reach B in the same amount of time, regardless of the position between A and B at which it starts sliding.
The inverted cycloid solves both the brachistochrone problem and the tautochrone problem.
How Do You Plot Parametric Curves?

One method is to eliminate the parameter. This gives the equation of a curve in the plane that contains the parametric curve.

Then go back to the parameter and investigate the motion of a point along the curve as you change parameter values.

You can use both algebraic and trigonometric identities to eliminate the parameter.
Problem 1.

- By eliminating the parameter, sketch the trajectory over the time interval $0 \leq t \leq 1$ of the particle whose parametric equations of motion are: $x = \cos(\pi t)$, $y = \sin(\pi t)$.
- Indicate the direction of motion and plot several points.

Solution: We have $x^2 + y^2 = 1$

So the motion takes place on a circle.

The particle goes from right to left over the top part of this circular arc.
**Problem 2.** By eliminating the parameter, sketch the curve and indicate the direction of increasing $t$.

\[ x = \sqrt{t}, y = 2t - 4, \quad 0 < t < \infty \]

**Solution:** Note that:

\[ x^2 = t \] and \[ \frac{y + 4}{2} = t \]. Thus \[ y = 2x^2 - 4 \] on the curve.

The curve therefore lies on the parabola:
We see that only non-negative values of $t$ are allowed, and as $t$ varies from 0 to $\infty$, the point on the curve goes from the bottom through all points on the right half of the parabola.