1. (3 pts each) Use the diagram on the right (showing areas) to find the following integrals. The diagram shows the graph of the function $f(x)$.

- (a) \[ \int_{2}^{4} 2f(x) \, dx = 2 \int_{2}^{4} f(x) \, dx = 14 \]
- (b) \[ \int_{1}^{2} f(x) \, dx = -\frac{1}{2} \int_{1}^{2} f(x) \, dx = -(-1.6) = 1.6 \]
- (c) \[ \int_{0}^{4} f(x) \, dx = 1 + 7 - 1.6 = 6.4 \]
- (d) \[ \int_{0}^{4} f(x) \, dx = 1 + 1.6 + 7 = 9.6 \]

2. A particle moves along a horizontal straight line. At any time $t > 0$ its acceleration is given by $a(t) = 8t$ meters/minute. Initial velocity and location are given by $v(0) = -4$ and $s(0) = 1$.

- (a) (8 pts) Find the formulas for the velocity $v(t)$ at any time $t > 0$ and the location $s(t)$ at any time $t > 0$.

\[ v(t) = \int 8t \, dt = 4t^2 + C \]
Since $v = -4$ when $t = 0$, $C = -4$. Thus $v(t) = 4t^2 - 4 = 4(t^2 - 1)$

\[ s(t) = \int 4t^2 - 4 \, dt = \frac{4t^3}{3} - 4t + C \]
Since $s = 1$ when $t = 0$, $C = 1$. Thus $s(t) = \frac{4t^3}{3} - 4t + 1$

- (b) (2 pts) At what time $t$ does the particle reverse direction?

$v = 0$ when $t = \pm 1$. We take $t = 1$ (since $-4$ is in the past).

- (c) (5 pts) What is the total distance traveled in the first two minutes?

\[ d = \int_{0}^{2} \sqrt{\left(4t^2 - 4\right)^2 + \left(4t^3 - 4t\right)^2} \, dt \]
\[ = \int_{0}^{2} \left[4t^2 - 4\right]^{1/2} \, dt + \int_{0}^{1} \left[4t^2 - 4\right]^{1/2} \, dt + \int_{1}^{2} \left[4t^2 - 4\right]^{1/2} \, dt \]
\[ = \left[ \frac{4}{3}t^3 - 4t \right]_{0}^{1} + \left[ \frac{4}{3}t^3 - 4t \right]_{1}^{2} = \left[ \frac{4}{3} - \frac{4}{3} \right] + \left[ \frac{32}{3} - 8 \right] - \left[ \frac{4}{3} - 4 \right] = 8 \]

3. Let $R$ be the region under the graph of $f(x) = x^3$ for $0 \leq x \leq 2$ and let $A$ be the area of $R$.

(a) (8 pts) Form the Riemann sum approximation of $A$, using 4 subintervals of equal length and right endpoints (see Figure). [Leave your answer in the form of an unevaluated sum.]

\[ S = \frac{1}{2} \left[ \frac{1}{2} \right]^3 + \frac{3}{2} \left[ \frac{3}{2} \right]^3 + 2 \left[ 2 \right]^3 = \frac{1}{2} \left[ \frac{1}{8} + \frac{27}{8} + 8 \right] = \frac{100}{16} \]
(b) (7 pts) Now find the exact area using the fundamental theorem of calculus.

\[ A = \int_0^2 x^2 \, dx = \frac{x^3}{4} \bigg|_0^2 = 4 \]

4. (15 pts) Solve the initial value problem \( \frac{dy}{dx} = 2\pi x \cos(\pi x^2) \); \( y(1) = 0 \)

\[ y = \int 2\pi x \cos(\pi x^2) \, dx \]

Let \( u = \pi x^2 \), \( du = 2\pi x \, dx \).

Then \( \int 2\pi x \cos(\pi x^2) \, dx = \int \cos(u) \, du = \sin(u) + C = \sin(\pi x^2) + C \).

Since \( 0 = y(1) = 0 + C, C = 0 \). Thus \( y = \sin(\pi x^2) \).

5. (14 pts each) Use substitution to evaluate the following.

(a) \( \int_0^2 \cos(x)e^{\sin(x)} \, dx \). Let \( u = \sin(x) \), \( du = \cos(x) \, dx \). Then \( \int_0^2 \cos(x)e^{\sin(x)} \, dx = \int_0^{\sin(2)} e^u \, du = e^u \bigg|_0^{\sin(2)} = e^{\sin(2)} - 1 \)

(b) \( \int_{\ln(2)}^{\ln(7)} \frac{\ln(x)^6}{x} \, dx \)

Let \( u = \ln(x) \), \( du = \frac{dx}{x} \). Then \( \int_{\ln(2)}^{\ln(7)} \frac{\ln(x)^6}{x} \, dx = \int u^6 \, du = \frac{u^7}{7} + C = \frac{\ln(7)^7}{7} + C \)

6. (15 pts) Find the area of the region bounded by the curves \( y = x^2 \), \( y = 2 - x \) (see figure)

For the limits of integration we have:

\( x^2 = 2 - x \); \( x^2 + x - 2 = 0 \); \( (x + 2)(x-1) = 0 \); \( x = -2, 1 \)

Then the area is

\[
\int_{-2}^{1} \left(2 - x - x^2\right) \, dx = \left[2x - \frac{x^2}{2} - \frac{x^3}{3}\right]_{-2}^{1} = \left(1 - \frac{5}{6}\right) - \left(-4 - \frac{8}{3}\right) = \frac{7}{6} + \frac{10}{3} = \frac{27}{6} = \frac{9}{2}
\]