Section 7.2. Find the volume of the solid that results when the region enclosed by the given curves is revolved about the x-axis.

6. \( y = \sec x, x = \pi/4, x = \pi/3, y = 0 \).

\[
V = \pi \int_{\pi/4}^{\pi/3} \sec^2 x \, dx = \pi(\sqrt{3} - 1)
\]
Section 7.2. Find the volume of the solid that results when the region enclosed by the given curves is revolved about the y-axis.

20. \( y = x^2 - 1, x = 2, y = 0. \)

\[
V = \pi \int_0^3 \left[ 2^2 - (y + 1) \right] dy \\
= \pi \int_0^3 (3 - y) dy = \frac{9\pi}{2}
\]
Section 7.3. Use the methods of cylindrical shells to find the volume of the solid that results when the region enclosed by the given curves is revolved about the y-axis.

6. \( y = \sqrt{x}, x = 4, x = 9, y = 0. \)

\[
V = \int_4^9 2\pi x (\sqrt{x}) \, dx
\]

\[
= 2\pi \int_4^9 x^{3/2} \, dx = \frac{844\pi}{5}
\]
Section 7.3. Use the methods of cylindrical shells to find the volume of the solid that results when the region enclosed by the given curves is revolved about the x-axis.

16. $xy = 4, x + y = 5.$

\[
V = \int_1^4 2\pi y(5 - y - 4/y)dy
\]

\[
= 2\pi \int_1^4 (5y - y^2 - 4)dy = 9\pi
\]
Section 7.4. Find the exact arc length of the given curve.

4. \[ x = \frac{1}{3} \left( \frac{3}{y^2 + 2} \right)^2 \text{ from } y = 0 \text{ to } y = 1 \]

\[
g'(y) = y(y^2 + 2)^{1/2}, \quad 1 + [g'(y)]^2 = 1 + y^2(y^2 + 2) = y^4 + 2y^2 + 1 = (y^2 + 1)^2,
\]

\[ L = \int_{0}^{1} \sqrt{(y^2 + 1)^2} dy = \int_{0}^{1} (y^2 + 1) dy = \frac{4}{3} \]
Section 7.4. Find the exact arc length of the given curve without eliminating the parameter.

10. \( x = (1 + t)^2; \quad y = (1 + t)^3 \quad (0 \leq t \leq 1) \)

\[
\frac{dx}{dt}^2 + \frac{dy}{dt}^2 = \left[2(1 + t)\right]^2 + \left[3(1 + t)^2\right]^2 = (1 + t)^2 \left[4 + 9(1 + t)^2\right],
\]

\[
L = \int_0^1 (1 + t)\left[4 + 9(1 + t)^2\right]^{1/2} dt = \left(80\sqrt{10} - 13\sqrt{13}\right)/27
\]
Section 7.5. Find the area of the surface generated by revolving the given curve about the x-axis.

4. \( x = \sqrt[3]{y}, (1 \leq y \leq 8) \)

\[
y = f(x) = x^3 \text{ for } 1 \leq x \leq 2, \quad f'(x) = 3x^2,
\]

\[
S = \int_1^2 2\pi x^3 \sqrt{1 + 9x^4} \, dx = \frac{\pi}{27} \left(1 + 9x^4\right)^{3/2} \Bigg|_1^2 = \frac{5\pi}{27} \left(29\sqrt{145} - 2\sqrt{10}\right) / 27
\]
Section 7.5. Find the area of the surface generated by revolving the given curve about the y-axis.

8. \( x = 2\sqrt{1-y}, 0 \leq y \leq -1 \)

\[ g'(y) = -\frac{1}{2}(1-y)^{-1/2}, \quad 1 + [g'(y)]^2 = \frac{2 - y}{1 - y}, \]

\[ S = \int_{-1}^{0} 2\pi (2\sqrt{1-y}) \sqrt{\frac{2-y}{1-y}} \, dy = 4\pi \int_{-1}^{0} \sqrt{2-y} \, dy = 8\pi (3\sqrt{3} - 2\sqrt{2})/3 \]
2. A variable force $F(x)$ in the $x$-direction is graphed in the accompanying figure. Find the work done by the force on a particle that moves from $x = 0$ to $x = 5$.

\[
W = \int_0^5 F(x) \, dx = \int_0^2 40 \, dx - \int_2^5 \frac{40}{3} (x - 5) \, dx = 80 + 60 = 140 \text{ J}
\]
Section 7.6

4. A spring whose natural length is 15 cm. exerts a force of 45 N when stretched to a length of 20 cm.
(a) Find the spring constant.
(b) Find the work that is done on stretching the spring 3 cm beyond its natural length.
(c) Find the work done in stretching the spring from a length of 20 cm to a length of 25 cm.

\[
\text{(a) } F(x) = kx, \quad F(0.05) = 0.05k = 45, \quad k = 900 \text{ N/m}
\]

\[
\text{(b) } W = \int_0^{0.03} 900x \, dx = 0.405 \text{ J}
\]

\[
\text{(c) } W = \int_{0.05}^{0.10} 900x \, dx = 3.375 \text{ J}
\]
Section 7.6

6. Assume that a force of 6N is required to compress a spring from a natural length of 4 m to a length of 31/2 m. Find the work required to compress the spring from its natural length to a length of 2 m.

\[ F(x) = kx, \quad F(1/2) = k/2 = 6, \quad k = 12 \text{ N/m}, \quad W = \int_0^2 12x \, dx = 24 \text{ J} \]
3. Find the fluid force against the surface shown.

\[ F = \int_0^2 62.4x(4)\,dx \]

\[ = 249.6 \int_0^2 x\,dx = 499.2 \text{ lb} \]
5. Find the fluid force against the surface shown.

\[ F = \int_0^5 9810x(2\sqrt{25 - x^2}) \, dx \]

\[ = 19,620 \int_0^5 x(25 - x^2)^{1/2} \, dx \]

\[ = 8.175 \times 10^5 \text{ N} \]