Using Integral Tables and Computer Algebra Systems

Tables of integrals have been available for many decades, and computer algebra systems have been available for almost 20 years. Integrals can be looked up in the table, or computed by the software. **In both cases, the user must be actively engaged.**
1. Tables do not have all possible integrals. They have most common forms, and include letters representing constants that may occur. The user must supply the constants, and in some cases must make substitutions to produce the desired form.

2. Computer algebra systems cannot integrate every integrable function (many functions cannot be integrated in closed form using functions whose name you know). Sometimes the user must make substitutions and possibly do some preliminary steps.

3. Computer algebra systems may produce an answer in awkward form, equivalent to the answer a human would get, but not obviously so. Also, these programs may ignore small things (such as absolute values) that result in an answer that is only partly true. Thus all computer algebra calculations should be checked in some way.
Example. Use tables and a C.A.S. to integrate the function
\[ \int \frac{x}{(2-3x)^2} \, dx \]

The closest form in the tables inside the book’s cover is
\[ \int \frac{u}{(a+bu)^2} \, du = \frac{1}{b^2} \left[ \frac{a}{a+bu} + \ln|a+bu| \right] + C \]

We let \( u = x, \) \( a = 2, \) and \( b = -3. \) Then
\[ \int \frac{x}{(2-3x)^2} \, dx = \frac{1}{9} \left[ \frac{2}{2-3x} + \ln|2-3x| \right] + C \]

We also check with Maple.
Example. Use tables and a C.A.S. to integrate the function

\[
\int \frac{dx}{x^2(1-5x)}
\]

The closest form in the tables inside the book’s cover is

\[
\int \frac{du}{u^2(a+bu)} = -\frac{1}{au} + \frac{b}{a^2} \ln \left| \frac{a+bu}{u} \right| + C
\]

We let \( u = x, \ a = 1, \) and \( b = -5. \) Then

\[
\int \frac{dx}{x^2(1-5x)} = -\frac{1}{x} - 5\ln \left| \frac{1-5x}{x} \right| + C
\]

We also check with Maple.
Example. Use tables and a C.A.S. to integrate the function
\[ \int \frac{\sqrt{4-x^2}}{x^2} \, dx \]

The closest form in the tables inside the book’s cover is
\[ \int \frac{\sqrt{a^2-u^2}}{u^2} \, du = -\sqrt{\frac{a^2-u^2}{u}} - \sin^{-1} \left( \frac{u}{a} \right) + C \]

We let \( u = x \), and \( a = 2 \). Then
\[ \int \frac{\sqrt{4-x^2}}{x^2} \, dx = -\sqrt{\frac{4-x^2}{x}} - \sin^{-1} \left( \frac{x}{2} \right) + C \]

We also check with Maple.
Example. Use tables and a C.A.S. to integrate the function

\[ \int \frac{\cos(4x)}{9 + \sin^2 4x} \, dx \]

The book’s tables have nothing that resembles this expression very closely. We realize, however, that it can be simplified by use of the substitution \( u = \sin 4x, \, du = 4 \cos 4x \, dx \). Then the integral becomes

\[ \int \frac{du}{4 \cdot 9 + u^2} \]

The tables have the expression

\[ \int \frac{du}{a^2 + u^2} = \frac{1}{a} \tan^{-1} \frac{u}{a} + C \]
In the above expression, we let $a = 3$. Then

$$\int \frac{du}{a^2 + u^2} = \frac{1}{a} \tan^{-1} \frac{u}{a} + C$$

so

$$\int \frac{du}{9 + u^2} = \frac{1}{3} \tan^{-1} \frac{u}{3} + C$$

$$\int \frac{\cos(4x)}{9 + \sin^2 4x} dx = \frac{1}{4} \int \frac{du}{9 + u^2} = \frac{1}{12} \tan^{-1} \frac{u}{3} + C = \frac{1}{12} \tan^{-1} \left( \frac{\sin 4x}{3} \right) + C$$

We also check with Maple.
Example. Use tables and a C.A.S. to integrate the function

\[ \int \cos(\sqrt{x}) \, dx \]

The book’s tables have nothing that resembles this expression very closely. We realize, however, that it can be simplified by use of the substitution \( u = \sqrt{x}, \, du = \frac{dx}{2\sqrt{x}} \). Then the integral becomes

\[ 2 \int u \cos u \, du \]

The tables have the expression

\[ \int u \cos u \, du = \cos u + u \sin u + C \]
\[ \int u \cos u \, du = \cos u + u \sin u + C \]

Then

\[ \int \cos(\sqrt{x}) \, dx = 2 \cos u + 2 u \sin u + C = 2 \cos \sqrt{x} + 2 \sqrt{x} \sin \sqrt{x} + C \]

We also check with Maple.