1. Consider the following mixing problem. Following what we demonstrated in our lecture you have to set up a matrix differential equation with the given initial condition to describe the behavior of the situation. We have three large tanks: $T_1$, $T_2$, and $T_3$. Each of the three tanks contains 5000 gallons of water in which initially 100 lb (Tank1) and 200 lb (Tank2) and 150 lb (Tank3) of fertilizer are dissolved. The circulations among the tanks are given as:

$T_1 \ x \ 8 \ \text{gal/min} \ \ T_2$, $T_2 \ x \ 3 \ \text{gal/min} \ \ T_1$, $T_1 \ x \ 5 \ \text{gal/min} \ \ T_3$, $T_3 \ x \ 10 \ \text{gal/min} \ \ T_1$, $T_1 \ x \ 5 \ \text{gal/min} \ \ T_2$, $T_3 \ x \ 10 \ \text{gal/min} \ \ T_2$, $T_2 \ x \ 5 \ \text{gal/min} \ \ T_3$

The mixture in each tank is kept uniform by stirring. Let $y_1(t)$ be the amount of fertilizer (in lb) in Tank1 at time $t$ (in minutes), $y_2(t)$ in Tank2 and $y_3(t)$ in Tank3. Note that the volume of solution in each tank remains the same in the course of time.

(a) Set up a matrix differential equation that governs the flow motion in the form: $y' = Ay$, where $y = \begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \end{bmatrix}$ and $A$ is a $3 \times 3$ matrix. Use Matlab to find the eigenvalues and the corresponding eigenvectors of matrix $A$ first.

(b) Use the theory in section 5.7 to find the unique solution for the initial value problem.

(c) How much fertilizer (in lb) is in Tank1 after 10 minutes?

Note that all calculation is proceeded using Matlab. Use % sign in place where a text is required.

2. Let $U$ be the $8 \times 4$ matrix given below. (a) Find the orthogonal projection of $y = (1, 1, 1, 1, 1, 1, 1, 1)$ in the column space of $U$.

(b) Find the shortest distance from $y$ to the column space of $U$. 
Note that the column vectors of $U$ are mutually orthogonal. You may want to use the theory in section 6.3, in particular, the *Orthogonal Decomposition Theorem* to do the problem in which the dot product of two vectors must be utilized. Define these vectors first and then call Matlab commends to carry out the calculation.

$$U = \begin{bmatrix}
-6 & -3 & 6 & 1 \\
-1 & 2 & 1 & -6 \\
3 & 6 & 3 & -2 \\
6 & -3 & 6 & -1 \\
2 & -1 & 2 & 3 \\
-3 & 6 & 3 & 2 \\
-2 & -1 & 2 & -3 \\
1 & 2 & 1 & 6 \\
\end{bmatrix}$$