Some Formulas:

\[ y = \log_a x \quad \Leftrightarrow \quad x = a^y \quad \ln(x) = \log_e x \quad (\text{where } e \approx 2.71828...) \quad \log_a x = \frac{\ln x}{\ln a} \]

1. (15 pts) Use the axes provided to sketch the curve whose parametric equations are: \( x = 2 \cos(t), \) \( y = -2 \sin(t), \) where \( 0 \leq t \leq \pi. \) Label important points on your graph, and indicate with an arrow the direction in which the curve is traced as the parameter increases.

**Solution.**

The curve is the lower half of the circle \( x^2 + y^2 = 4. \)

As the parameter \( t \) increases from 0 to \( \pi, \) the point \( (x,y) \) traverses the semi-circle in the clockwise direction from the point \( (2,0) \) to the point \( (-2,0). \)

2. Find the exact value of each of the following: (5 pts each)

(a) \( \log_{10}25 + \log_{10}8 - \log_{10}2 \)

**Solution.**

\[
\log_{10}25 + \log_{10}8 - \log_{10}2 = \log_{10}\left(\frac{25 \times 8}{2}\right) = \log_{10}(100) = \log_{10}(10^2) = 2
\]

(b) \( e^{2\ln(2)} \)

**Solution.**

\[
e^{2\ln(2)} = e^{\ln(2^2)} = e^{\ln(4)} = 4
\]
3. (10 pts) Use the axes provided to graph the function \( y = -\ln(x+1) \). Label the axes and important points (such as intercepts) on your graph.

**Solution.**

Start with the graph of \( y = \ln(x) \), then shift it one unit to the left and reflect (flip) it about the \( x \)-axis.

4. Under warm and damp conditions a certain mosquito population is known to double in size every 6 hours. Suppose that there are initially 10 mosquitoes in the population.

(a) (5 pts) What is the size of the population after 24 hours?

**Solution.**

After 24 hours the population is \( 10 \times 2^4 = 160 \) (population doubles 4 times in 24 hours).

(b) (5 pts) Find a formula for the size of the population after \( t \) hours.

**Solution.**

The population doubles every 6 hours. Therefore, after \( t \) hours, the population is \( P(t) = 10 \times 2^{\frac{t}{6}} \).

(c) (5 pts) How long (hours) will it take for the population to reach 1000 mosquitoes? Express your answer in exact form (as a logarithm), then use your calculator to produce a decimal approximation.

**Solution.**

Solve the equation \( P(t) = 1000 \) for \( t \).

This leads to \( 10 \times 2^{\frac{t}{6}} = 1000 \), so \( 2^{\frac{t}{6}} = 100 \) and \( \frac{t}{6} = \log_2 100 = \frac{\ln(100)}{\ln(2)} \).

Thus \( t = 6 \log_2(100) = 6 \frac{\ln(100)}{\ln(2)} \approx 39.86 \) hours.