Preface. This assignment is a term project (the one and only MFE III Special Assignment) that will result in a report on the cycloid, a famous curve that we have alluded to several times in our MFE lectures this year. Recall that the cycloid is the curve traced out by a point P on the circumference of a rolling wheel.

If the wheel has radius a (use r if you prefer), and rolls along the x-axis with the point P starting at the origin, then the curve can be described by the parametric equations

\[ x = a(\theta - \sin \theta) \quad y = a(1 - \cos \theta) \]

where the parameter \( \theta \) (use another variable if you prefer) corresponds to the angle through which the wheel has been rotated. The derivation of these equations can be found on p51 in your text. Note that one arch of the cycloid is generated by one rotation of the wheel and thus corresponds to \( 0 \leq \theta \leq 2\pi \).

Guidelines on Group Work. You are to work on this assignment in teams of four to six students, not necessarily from the same recitation section. You are free to form your own teams, but I want to know in advance (early in the term) who your team members are. I also want you to choose a team captain (or co-captains) so that there will be a designated representative with whom I can more readily communicate. Note that working in a team is one of the requirements of this assignment (not an option). Each team will submit one paper only, and all members of the team will receive the same grade on the assignment.

Discussing the assignment with other students, or other teams of students, is permissible. However, when it comes time to prepare the paper that you will be submitting you should do so only within your team, and you should not share that work with others or use the work of others. We will not accept papers where (in our best judgment) this code has been violated.

Due Date. A brief progress report will be required during the third week of the term, and the completed assignment will be due the eighth week. Late papers will not be accepted.

The Assignment. One of the purposes of this project is for you to get some experience, as a group, in preparing a nice report with significant technical content. Thus your paper will be evaluated on style as well as substance. We expect the work to be well-organized and nicely formatted, and for the written part to be grammatically correct (and spell-checked). The assignment itself consists of several parts, each of which should constitute a section of your report.
PART 1: GEOMETRIC CALCULATIONS. Perform (with Maple) the following calculations pertaining to the cycloid:

(a) Area of the region \( \mathcal{A} \) under one arch of the cycloid.
(b) Arc length of one arch.
(c) Volume of the solid obtained when the region \( \mathcal{A} \) is rotated about the x-axis.
(d) Volume of the solid obtained when the region \( \mathcal{A} \) is rotated about the line \( x = a\pi \).
(e) Centroid of the region \( \mathcal{A} \).

Note that these calculations should be done symbolically (with Maple), and that your answers will be in terms of \( a \) (or \( r \)). Thus the end result will be formulas for these various geometric quantities in terms of the radius of the wheel.

PART 2: THE BRACHISTRONE PROPERTY. Suppose now that we reorient the coordinate axes so that the y-axis points downward. Then the parametric equations still apply, but the resulting curve (inverted cycloid) looks like this:

![Inverted Cycloid]

This curve has two interesting physical properties. The first relates to the origin O and the point B at the bottom of the first arch. Among all smooth curves joining these points, the cycloid is the curve along which a frictionless bead, subject only to the force of gravity, will slide in the least time. This makes the cycloid a **brachistochrone** ("brah-kiss-toe-krone"), or **shortest-time** curve for these two points. Although the proof of this fact is beyond the scope of this project, we can (with Maple) perform some calculations that illustrate and lend credence to this statement.

- additional information and instructions will follow -

PART 3: THE TAUROCROME PROPERTY. A second property of the inverted cycloid is that even if you start the bead partway down the curve toward the point B, it will still take the bead the same amount of time to reach B. This makes the cycloid a **taurochrone** ("taw-toe-chron"), or **same-time** curve for O and B. The following calculations (with Maple) will illustrate this fact.

- additional information and instructions will follow -

PART 4: This part of the assignment (and possibly a Part 5), is still under development at this time and will be reserved for a subsequent announcement.