Part I. The problems on this part are of multiple-choice type. Circle the correct answer.

1. Let \( A \) be the area of the region under the graph of \( y = 25 - x^2 \) for \( 0 \leq x \leq 4 \). Find the value of the approximation to \( A \) obtained by using four rectangles of equal width and right endpoints (see figure).
   
   \[ \text{A) 65 B) 68 C) 70 D) 72 E) 75 F) 78 G) 80 H) 82} \]

**Solution.** The right endpoint approximation with \( n = 4 \) is:

\[
A \approx [(25-1)+(25-4)+(25-9)+(25-16)](1) = 24 + 21 + 16 + 9 = 70
\]

The correct answer is C.

2. Find the exact value of the area of the region under the graph of \( y = 25 - x^2 \) for \( 0 \leq x \leq 4 \).
   
   \[ \text{A) 65 B) 67 \frac{1}{2} C) 70 D) 72 \frac{1}{3} E) 75 F) 78 \frac{2}{3} G) 80 H) 82 \frac{1}{2} } \]

**Solution.** The exact value of the area is

\[
A = \int_0^4 (25 - x^2)dx = (25x - \frac{x^3}{3}) \bigg|_0^4 = 100 - \frac{64}{3} = \frac{236}{3}.
\]

The correct answer is F.

3. Find the value of the integral \( \int_{-2}^1 |x + 1| \, dx \).
   
   \[ \text{A) -1 B) 0 C) 1 D) } \]
   
   \[ \text{E) -} \frac{1}{2} \text{ F) } \frac{1}{2} \text{ G) } \frac{3}{2} \text{ H) } \frac{5}{2} \]

**Solution.** The easiest way to evaluate this integral is to interpret it in terms of areas (see figure):

\[
\int_{-2}^1 |x + 1| \, dx = A_1 + A_2 = \frac{1}{2} + 2 = \frac{5}{2}
\]

The correct answer is H.
4. If \( \int_0^2 f(x) \, dx = 3 \) and \( \int_0^2 g(x) \, dx = -2 \), what is the value of \( \int_0^2 [2f(x) - 3g(x)] \, dx \)?

A) 2    B) -2    C) 3    D) -3
E) 6    F) -6    G) 12    H) -12

**Solution.** From the linearity of integration we have:

\[
\int_0^2 [2f(x) - 3g(x)] \, dx = 2\int_0^2 f(x) \, dx - 3\int_0^2 g(x) \, dx = 2(3) - 3(-2) = 12
\]

The correct answer is G.

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**Part II. The remaining problems are to be worked out in detail. You must show all your work on this paper to receive full credit.**

5. A particle is moving along a line with velocity function \( v(t) = 4 - 2t \) (meters per second). Find the total distance (back and forth) traveled during the time interval \( 0 \leq t \leq 6 \).

**Solution.** The total distance (back and forth) is given by:

\[
\int_0^6 |v(t)| \, dt = \int_0^2 (4 - 2t) \, dt + \int_2^6 (2t - 4) \, dt = (4t - t^2) \bigg|_0^2 + (t^2 - 4t) \bigg|_2^6 = (8 - 4) + [(36 - 24) - (4 - 8)] = 20
\]

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6. Use the Substitution Rule (Change of Variable) to evaluate each of the following integrals. All steps must be shown to receive credit.

(a) \( \int \frac{x^2}{\sqrt{2 + x^3}} \, dx \)

(b) \( \int \frac{1}{(2x + 1)^2} \, dx \)
Solutions.

(a) Let \( u = 2 + x^3 \) (\( \therefore \ du = 3x^2\,dx \)).

Then \( \int \frac{x^2}{\sqrt{2 + x^3}} \, dx = \frac{1}{3} \int \frac{1}{\sqrt{2 + x^3}} \, 3x^2\,dx = \frac{1}{3} \int \frac{1}{\sqrt{u}} \, du = \frac{1}{3} \frac{1}{\frac{3}{2}} \sqrt{2 + x^3} + C = \frac{2}{3} \sqrt{2 + x^3} + C \).

(b) Let \( u = 2x + 1 \) (\( \therefore \ du = 2dx \)).

Then \( \int \frac{1}{(2x + 1)^2} \, dx = \frac{1}{2} \int \frac{1}{u^2} \, du = \frac{1}{2} \left(-\frac{1}{u}\right) + C = -\frac{1}{2} \frac{1}{2x + 1} + C \).

7. (8 pts) Find the derivative of the function \( g(x) = \int_0^x \frac{1}{\sqrt{1 + t^4}} \, dt \).

Solution. Using the Fundamental Theorem of Calculus (Part 1), together with the Chain Rule:

\[
g'(x) = \frac{d}{dx} \left[ \int_0^x \sqrt{1 + t^4} \, dt \right] = \sqrt{1 + (\sqrt{x})^4} \frac{d}{dx} \sqrt{x} = \sqrt{1 + x^2} \frac{1}{2\sqrt{x}} = \frac{1}{2} \frac{1}{x} \sqrt{1 + x^2}
\]

8. Use a change of variable, together with a formula from the table below, to find the exact value of the integral \( \int_0^{\pi/2} \cos(x) \sqrt{4 + \sin^2 x} \, dx \). Show all steps. A numerical answer alone will receive no credit.

Solution. Let \( u = \sin(x) \) (\( \therefore \ du = \cos(x)\,dx \), \( u = 0 \) when \( x = 0 \), and \( u = 1 \) when \( x = \frac{\pi}{2} \)).

Then \( \int_0^{\pi/2} \cos(x) \sqrt{4 + \sin^2 x} \, dx = \int_0^1 \sqrt{4 + u^2} \, du = \left[ \frac{u}{2} \sqrt{4 + u^2} + \frac{4}{2} \ln\left(u + \sqrt{4 + u^2}\right) \right]_0^1 \)

\[
= \left[ \frac{1}{2} \sqrt{5} + 2 \ln(1 + \sqrt{5}) \right] - \left[ 0 + 2 \ln(2) \right] = \frac{1}{2} \sqrt{5} + 2 \ln\left(\frac{1 + \sqrt{5}}{2}\right)
\]

\[
\begin{align*}
\int & \frac{\sqrt{a^2 + u^2}}{u} \, du = \sqrt{a^2 + u^2} - a \ln \left| \frac{a + \sqrt{a^2 + u^2}}{u} \right| + C \\
\int & \frac{a^2 + u^2}{u} \, du = \frac{u}{2} \sqrt{a^2 + u^2} + \frac{4}{2} \ln\left(u + \sqrt{a^2 + u^2}\right) + C \\
\int & \frac{u^2}{\sqrt{a^2 + u^2}} \, du = -\frac{\sqrt{a^2 + u^2}}{u} + \ln\left(u + \sqrt{a^2 + u^2}\right) + C
\end{align*}
\]
9. (12 pts) Find the exact value of the integral \( \int_0^1 xe^{2x} \, dx \). Show all steps. A numerical answer alone will receive no credit.

**Solution.** Use integration by parts with \( u = x \) and \( dv = e^{2x} \, dx \left( \therefore du = dx \text{ and } v = \frac{1}{2} e^{2x} \right) \).

\[
\int_0^1 xe^{2x} \, dx = \frac{1}{2} xe^{2x} \bigg|_0^1 - \frac{1}{2} \int_0^1 e^{2x} \, dx = \frac{1}{2} xe^{2x} \bigg|_0^1 - \frac{1}{4} e^{2x} \bigg|_0^1 = \frac{1}{2}(e^2 - 0) - \frac{1}{4}(e^2 - 1) = \frac{e^2 + 1}{4}
\]

10. (6 pts) Write out the form of the partial fraction decomposition of \( f(x) = \frac{x^2 - 3x + 4}{x^3(x + 2)(x^2 + 4)} \). DO NOT ATTEMPT TO FIND THE UNKNOWN CONSTANTS.

**Solution.** The form of the decomposition is:

\[
\frac{x^2 - 3x + 4}{x^3(x + 2)(x^2 + 4)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x + 2} + \frac{D}{x^2 + 4}
\]

11. (12 pts) Use partial fractions to evaluate the integral \( \int \frac{2 + x}{(x - 1)(x + 3)} \, dx \).

**Solution.** The integrand has a partial fraction decomposition of the form

\[
\frac{2 + x}{(x - 1)(x + 3)} = \frac{A}{x - 1} + \frac{B}{x + 3}
\]

where \( x + 2 = A(x + 3) + B(x - 1) = (A + B)x + (3A - B) \). It follows that \( A + B = 1 \) and \( 3A - B = 2 \); thus \( A = \frac{3}{4} \) and \( B = \frac{1}{4} \). The integral can then be calculated as follows:

\[
\int \frac{2 + x}{(x - 1)(x + 3)} \, dx = \frac{3}{4} \int \frac{1}{x - 1} \, dx + \frac{1}{4} \int \frac{1}{x + 3} \, dx = \frac{3}{4} \ln|x - 1| + \frac{1}{4} \ln|x + 3| + C
\]