Part I. The problems on this part are of multiple-choice type. Circle the correct answer.

1. The integral \( A(x) = \int_0^x e^{-\sqrt{t}} \, dt \) represents the area of the region under the graph of \( y = e^{-\sqrt{t}} \) for \( 0 \leq t \leq x \) (see figure). Given this fact, what is the rate of change of \( A(x) \) with respect to \( x \) at \( x = 1 \)?

A) \( e \) \quad B) \( -e \) \quad C) \( \sqrt{e} \) \quad D) \( -\sqrt{e} \)

E) 1 \quad F) -1 \quad G) \frac{1}{e} \quad H) -\frac{1}{e}

Solution. From the Fundamental Theorem of Calculus (Part 1), the derivative of \( A \) is \( A'(x) = e^{-\sqrt{x}} \); thus \( A'(1) = e^{-1} = \frac{1}{e} \). The correct answer is G.

2. If \( \int_0^1 f(x) \, dx = 7 \) and \( \int_0^3 f(x) \, dx = 6 \), what is the value of \( \int_1^3 f(x) \, dx \)?

A) 0 \quad B) 1 \quad C) 2 \quad D) 3

E) -1 \quad F) -2 \quad G) -3 \quad H) 13

Solution. \( \int_1^3 f(x) \, dx = \int_0^3 f(x) \, dx - \int_0^1 f(x) \, dx = 6 - 7 = -1 \). The correct answer is E.

3. The graph of \( f \) is as shown in the figure. Evaluate the integral \( \int_0^{10} f(x) \, dx \) by interpreting it in terms of areas.

A) 2 \quad B) 3 \quad C) 4

D) \frac{5}{2} \quad E) \frac{7}{2} \quad F) \frac{9}{2}

G) 13 \quad H) 22

Solution. The portion of the graph from 2 to 10 is symmetric with the x-axis. Thus \( \int_2^{10} f(x) \, dx = 0 \), and so \( \int_0^{10} f(x) \, dx = \int_0^2 f(x) \, dx = 4 \) (area of a trapezoid). The correct answer is C.
Part II. These problems are to be worked out in detail. You must show all your work on this paper to receive full credit.

4. Use the Midpoint Rule with \( n = 3 \) to approximate the integral \( \int_0^2 x^2 \, dx \). Leave your answer in the form of an unevaluated sum. [You are not required to add the terms or to compute a decimal form.] Use the axes provided to sketch the approximating rectangles.

**Solution.** With \( n = 3 \) the three subintervals are \([0, \frac{2}{3}],[\frac{2}{3}, \frac{4}{3}],[\frac{4}{3}, 2]\) with midpoints \( x_1 = \frac{1}{3}, x_2 = 1, \) and \( x_3 = \frac{5}{3} \). Thus \( \Delta x = \frac{2}{3} \) and the Midpoint Rule approximation of the integral is

\[
\int_0^2 x^2 \, dx \approx \left( \left( \frac{1}{3} \right)^2 + (1)^2 + \left( \frac{5}{3} \right)^2 \right) \left( \frac{2}{3} \right) = \frac{70}{27} \approx 2.593
\]

The approximating rectangles are as shown in the figure.

5. Evaluate the following integrals exactly using the Fundamental Theorem of Calculus. All steps must be shown. A numerical answer alone will be worth no points.

**Solutions.**

(a) \( \int_2^5 \frac{1}{x} \, dx = \ln(x) \bigg|_2^5 = \ln(5) - \ln(2) = \ln\left(\frac{5}{2}\right) \approx 0.9163 \)

(b) \( \int_{\frac{\pi}{6}}^{\pi} \sin x \, dx = -\cos(x) \bigg|_{\frac{\pi}{6}}^{\pi} = -\cos(\pi) + \cos\left(\frac{\pi}{6}\right) = -(-1) + \frac{\sqrt{3}}{2} = 1 + \frac{\sqrt{3}}{2} \approx 1.866 \)