RECTILINEAR MOTION

Consider an object moving along a straight line (car moving along a straight highway, rock falling vertically, mass attached to a vibrating spring, etc.)

Call the straight line the "S-axis" and let the object's S-coordinate at time \( t \) be denoted \( s(t) \).

\[
\begin{array}{cccccc}
0 & & & & & \text{s(t)} \\
\end{array}
\]

The graph of the function \( s(t) \) in the \( ts \)-plane is the position versus time curve of the object.

E.G.,
**INTERPRETATION:**

\[ t = 0 \text{ to } t = a \]
Object is to the left of the origin and moving toward the origin.

\[ t = a \text{ to } t = c \]
To right of the origin, moving away from it.

\[ t = c \text{ to } t = e \]
Turns around and heads back toward the origin.

\[ t = e \]
Comes to a stop before arriving at the origin and stays there.

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**INSTANTANEOUS VELOCITY**

\[ v(t) = s'(t) = \text{slope of tangent to } s(t) \]

\[ v(t) > 0 \]
Moving to the right (position curve increasing)

\[ v(t) < 0 \]
Moving to the left (position curve decreasing)
INSTANTANEOUS SPEED : $|n(t)|$

The graph of the function $n(t)$ in the $tn$-plane is the velocity versus time curve of the object.

E.G.,

INTERPRETATION :

$t = 0$ to $t = b$  
Velocity starts out positive and becomes larger and larger until it reaches a maximum value at $t = b$.

$t = b$ to $t = c$  
Velocity decreases from its maximum positive value at $t = b$ to zero at $t = c$.
\[ t = c \text{ to } t = d \] Velocity starts out at zero and becomes increasingly negative until it reaches its most negative value at \( t = d \)

\[ t = d \text{ to } t = e \] Velocity becomes less and less negative until it reaches the value zero at \( t = e \)

\[ t > e \] Velocity is zero

Thus, starting from its position to the left of the origin at \( t = 0 \), it heads toward the origin at increasing speed, zooms past the origin at \( t = a \), continues to pick up speed until \( t = b \)

When it starts to slow down, coming to an instantaneous stop at \( t = c \) so that it can turn around and head back toward the origin, increasing its speed until \( t = d \) when it starts to slow down, coming to a complete stop at \( t = e \) before arriving at the origin.

**Instantaneous Acceleration:** \( a(t) = v'(t) = s''(t) \)

- \( a(t) > 0 \) : Velocity increasing (Graph of \( s(t) \) is concave up)
- \( a(t) < 0 \) : Velocity decreasing (Graph of \( s(t) \) is concave down)
NOTE THAT

\[ a(t) \text{ and } \ddot{a}(t) \text{ have the same sign } \Rightarrow \text{ speeding up} \]

\[ a(t) \text{ and } \ddot{a}(t) \text{ have opposite signs } \Rightarrow \text{ slowing down} \]

\[ S(t) = -2t^3 - 23t^2 + 40t + 3 \]

**EXAMPLES:**

1. **ANALYZE THE NOTION OF AN OBJECT WHOSE POSITION FUNCTION IS**

   \[ S(t) = -2t^3 - 23t^2 + 40t + 3 \]

   **FOR** \( t > 0 \),

   \[ a(t) = S'(t) = -6t^2 - 42t + 40 = 6(t^2 - 7t + 10) = 6(t-2)(t-5) \]

   \[ = 0 \text{ when } t = 2, 5 \]

   \[ a(t) = S''(t) = 12t - 42 = 6(t - 7) \]

   \[ = 0 \text{ when } t = \frac{7}{2} \]
\[ s(t) = -\frac{1}{2} gt^2 + v_0 t + s_0 \] (we will prove this shortly)

WHERE \( s_0 \) IS THE HEIGHT AT \( t = 0 \),
\( v_0 \) IS THE VELOCITY AT \( t = 0 \) AND
\( g \) IS A CONSTANT (32 \text{ ft/sec}^2 \text{ OR}
9.8 \text{ m/sec}^2 \)

\[ v(t) = s'(t) = -gt + v_0 \]
\[ a(t) = v'(t) = -g \] (constant acceleration)

(a) A ROCK, DROPPED FROM AN UNKNOWN HEIGHT, STRIKES THE
GROUND WITH A SPEED OF 24 \text{ m/sec}. FIND THIS UNKNOWN
INITIAL HEIGHT.