1. Let X be a continuous random variable whose density is:

\[
f(x) = \begin{cases} 
3x^2 & 0 \leq x \leq 1 \\
0 & \text{elsewhere}
\end{cases}
\]

Let Y = X^2.

(a) Find the cdf of Y. **Solution.** Since X is zero outside [0,1], so is Y. Thus if F_Y(y) is the cdf of Y, it must be 0 for y≤0 and 1 for y≥1. If 0<y<1 the \( F_Y(y) = P[Y \leq y] = P[X^2 \leq y] \):

\[
P\left[ X \leq \sqrt{y} \right] = \int_0^{\sqrt{y}} 3x^2 \, dx = \left( \sqrt{y} \right)^3.
\]

(b) **Solution.** Find the pdf of Y. \( f_Y(y) = \frac{df_Y(y)}{dy} = \frac{3}{2\sqrt{y}} \).

(c) **Solution.** \( P\left[ \frac{1}{2} < Y < \frac{3}{4} \right] = \left( \frac{3}{4} \right)^{\frac{3}{2}} - \left( \frac{1}{2} \right)^{\frac{3}{2}} \)

2. A box contains 5 keys of which only one will open a particular door. Keys are drawn without replacement and tried on the door until it is opened. Let X be the number of tries required to select the correct key. Determine E(X) and V(X).

**Solution.** \( p_1 = P[X=1] = \frac{1}{5} \), \( p_2 = P[X=2] = \frac{4}{5} \cdot \frac{1}{4} = \frac{1}{5} \). Continuing we see \( P[X=\frac{1}{5}] \) for \( k = 1,2,3,4,5 \).

E(X) = \( \frac{1}{5} [1+2+3+4+5] = 3 \). E(X^2) = \( \frac{1}{5} [1+4+9+16+25] = 11 \), and so V(X) = 11−9=2.

3. Suppose that X and Y are random variables with joint pdf

\[
f(x,y) = 6x^2y, \quad 0 < x < 1, \quad 0 < y < 1.
\]

(a) Determine the individual pdfs of X and Y.

**Solution.** \( f_X(x) = \int_0^1 6x^2y \, dy = 3x^2 \) if \( 0<x<1 \). \( f_Y(y) = \int_0^1 6x^2 \, dx = 2y \) if \( 0<y<1 \)

(b) What is the probability that Y < X^2?

**Solution.** \( P[Y < X^2] = \int_0^1 \int_0^{x^2} 6x^2y \, dy \, dx = \frac{3}{7} \).
4. Suppose the average height $H$ of a U.S. female is known to be a normal random variable with mean 62 in. and variance 9.

(a) What is the probability that a randomly chosen female will have height less than 5 ft. 4 inches?

**Solution.**

\[
P[H < 64] = P \left[ \frac{H - 62}{3} < \frac{64 - 63}{3} = \frac{2}{3} \right] = 0.7454 \text{ from tables}
\]

(b) Find the height $h_0$ (in inches), if we know that the probability of a randomly chosen female having height less that $h_0$ is .72?

**Solution.**

\[
P[H < h_0] = P \left[ \frac{H - 62}{3} < \frac{h_0 - 62}{3} \right] = 0.72. \text{ From table, } \frac{h_0 - 62}{3} = 0.58. \text{ Thus }
\]

\[
h_0 = 62 + 3(0.58) = 63.74.
\]

5. Telephone calls for technical support arrive at a software company randomly in time during the first hour of service each day. The average number of calls in this hour is 10.

(a) What is the probability of receiving 2 or fewer calls?

**Solution.**

$N$ is Poisson with $\lambda = 10$. Thus

\[
P[N = 0] + P[N = 1] + P[N = 2] = e^{-10} \left[ 1 + 10 + \frac{50}{2} \right] = 61e^{-10} = p
\]

(b) If a particular technical support operator works the first hour 10 times in a month, what is the probability that he or she will receive 2 or fewer calls exactly three times?

**Solution.**

\[
\text{Prob} = \binom{10}{3} p^3 (1-p)^7 = 120(61)^3 e^{-30}(1-61e^{-10})^7.
\]