PX-10-415

TABLE 6-1

EXTRACTION OF SQUARE ROOTS BY THE DIVIDER AND SQUARE ROOTER – Period II

PROBLEM: To find $\sqrt{R}$ where $R = N^2 + \varepsilon$

Assume $N^2 = 10^{18}a_0^2 + 10^{17}(2a_0a_1) + 10^{16}(2a_0a_2 + a_1^2) + 10^{15}(2a_0a_3 + 2a_1a_2) + 10^{14}(2a_1a_3 + a_2^2) + \cdots + a_9^2$

So that* $N = 10^9 a_0 + 10^8 a_1 + 10^7 a_2 + \cdots + a_9$ where the a_i are integers between 0 and 9

When the square rooting commences, the numerator accumulator holds N.

OPERATION PERFORMED ON CONTENTS OF NUMERATOR ACCUMULATOR	REMAINDER IN NUMERATOR ACCUMULATOR AS A RESULT OF OPERATION IN COLUMN 1.	CONTENTS OF DENOMINATOR (TWO-ROOT) ACCUMULATOR
In basic square rooting sequence before first overdraft, SUBTRACT $10^{18}\sum_{i=1}^{a_0+1}(2i-1) = 10^{18}(a_0^2 + 2a_0 + 1)$	$-10^{18}(2a_0+1) + 10^{17}(2a_0a_1) + 10^{16}(2a_0a_2 + a_1^2) + \cdots + a_9^2 + \varepsilon$	AFTER OVERDRAFT OCCURS BUT BEFORE SHIFT SEQUENCE $10^8[2(a_0+1) + 1]$ AT END OF FIRST ADDITION TIME OF SHIFT SEQUENCE $10^8[2(a_0+1)]$ AT END OF SHIFT SEQUENCE $10^8[2(a_0+1)] - 10^7$
After first shift sequence, but before second overdraft ADD $10^{17}[2(a_0+1)][10-a_1] - 10^{16}\sum_{i=1}^{10-a_1}(2i-1)$ $= 10^{18}(2a_0+1) - 10^{17}(2a_0a_1) - 10^{16}a_1^2$	$10^{16}(2a_0a_2) + 10^{15}(2a_0a_3 + 2a_1a_2) + \cdots + a_9^2 + \varepsilon$	AFTER OVERDRAFT OCCURS BUT BEFORE SHIFT SEQUENCE $10^8[2(a_0+1)] - 10^7[2(10-a_1) + 1]$ $= 2[10^8a_0 + 10^7a_1] - 10^7$ AT END OF FIRST ADDITION TIME OF SHIFT SEQUENCE $2[10^8a_0 + 10^7a_1]$ AT END OF SHIFT SEQUENCE $2[10^8a_0 + 10^7a_1] + 10^6$

*. Compare N with the column showing the contents of the denominator accumulator and note the displacement of the answer. (See Sec. 6.4.3.)