CS 430
Computer Graphics

Polygon Clipping and Filling
Week 3, Lecture 5

David Breen, William Regli and Maxim Peysakhov
Department of Computer Science
Drexel University
Outline

• Polygon clipping
  – Sutherland-Hodgman,
  – Weiler-Atherton
• Polygon filling
  – Scan filling polygons
  – Flood filling polygons
• Introduction and discussion of homework #2
• Ordered set of vertices (points)
  – Usually counter-clockwise
• Two consecutive vertices define an edge
• Left side of edge is inside
• Right side is outside
• Last vertex implicitly connected to first
• In 3D vertices should be co-planar
Polygon Clipping

• Lots of different cases

• Issues
  – Edges of polygon need to be tested against clipping rectangle
  – May need to add new edges
  – Edges discarded or divided
  – Multiple polygons can result from a single polygon
The Sutherland-Hodgman Polygon-Clipping Algorithm

• Divide and Conquer

• Idea:
  – Clip single polygon using single infinite clip edge
  – Repeat 4 times

• Note the generality:
  – 2D convex n-gons can clip arbitrary n-gons
  – 3D convex polyhedra can clip arbitrary polyhedra
Sutherland-Hodgman Algorithm

• Input:
  – \( v_1, v_2, \ldots, v_n \) the vertices defining the polygon
  – Single infinite clip edge w/ inside/outside info

• Output:
  – \( v'_1, v'_2, \ldots, v'_m \), vertices of the clipped polygon

• Do this 4 (or \( n_e \)) times

• Traverse vertices (edges)

• Add vertices one-at-a-time to output polygon
  – Use inside/outside info
  – Edge intersections
Sutherland-Hodgman Algorithm

- Can be done incrementally
- If first point inside add. If outside, don’t add
- Move around polygon from $v_1$ to $v_n$ and back to $v_1$
- Check $v_i, v_{i+1}$ wrt the clip edge
- Need $v_i, v_{i+1}$’s inside/outside status
- Add vertex one at a time. There are 4 cases:

1994 Foley/VanDam/Finer/Huges/Phillips ICG
Sutherland-Hodgman Algorithm

• Given polygon $P$ \quad $P' = P$
  
  – foreach clipping edge (there are 4) {
    • Clip polygon $P'$ to clipping edge
      
      – foreach edge in polygon $P'$
        
        » Check clipping cases (there are 4)
          
          » Case 1 : Output $v_{i+1}$ to $P''$
          
          » Case 2 : Output intersection point to $P''$
          
          » Case 3 : No output
          
          » Case 4 : Output intersection point & $v_{i+1}$ to $P''$

    • $P' = P''$
  }

Sutherland-Hodgman Algorithm

\{A, B, C, D, E, F, G, H, A\}

Animated by Max Peysakhov @ Drexel University
Final Result

Note: Edges XY and ZW!
Issues with Sutherland-Hodgman Algorithm

- Clipping a concave polygon
- Can produce two CONNECTED areas
Weiler-Atherton Algorithm

- General clipping algorithm for concave polygons with holes
- Produces multiple polygons (with holes)
- Make linked list data structure
- Traverse to make new polygon(s)
Weiler-Atherton Algorithm

- Given polygons A and B as linked list of vertices (counter-clockwise order)
- Find all edge intersections & place in list
- Insert as “intersection” nodes
- Nodes point to A & B
- Determine in/out status of vertices
Linked List Data Structure

Intersection Nodes
Intersection Special Cases

- If “intersecting” edges are parallel, ignore
- Intersection point is a vertex
  - Vertex of A lies on a vertex or edge of B
  - Edge of A runs through a vertex of B
  - Replace vertex with an intersection node
Weiler-Atherton Algorithm: Union

- Find a vertex of A outside of B
- Traverse linked list
- At each intersection point switch to other polygon
- Do until return to starting vertex
- All visited vertices and nodes define union’ed polygon
Example: Union

\{V1, V2, V3, P0, V8, V4, P3, V0\}, \{V6, P1, P2\}
Example
Result
Weiler-Atherton Algorithm: Intersection

- Start at intersection point
  - If connected to an “inside” vertex, go there
  - Else step to an intersection point
  - If neither, stop
- Traverse linked list
- At each intersection point switch to other polygon and remove intersection point list
- Do until return to starting intersection point
- If intersection list not empty, pick another one
- All visited vertices and nodes define and’ed polygon
Example: Intersection

\{P1, V7, P0\}, \{P3, V5, P2\}
Boolean Special Cases

If polygons don’t intersect

– Union
  • If one inside the other, return polygon that surrounds the other
  • Else, return both polygons

– Intersection
  • If one inside the other, return polygon inside the other
  • Else, return no polygons
Point P Inside a Polygon?

- Connect P with another point P' that you know is outside polygon
- Intersect segment PP' with polygon edges
- Watch out for vertices!
- If # intersections is even (or 0) → Outside
- If odd → Inside
Point P Inside a Rectangle?

- Just re-use code from Cohen-Sutherland algorithm
- If a vertex’s code equals zero, it’s inside
- Else, it’s outside
Edge clipping

• Re-use line clipping from HW1
  – Similar triangles method
  – Cyrus-Beck line clipping
• Yet another technique
Intersecting Two Edges (1)

- Edge 0 : \((P_0,P_1)\)
- Edge 2 : \((P_2,P_3)\)
- \(E_0 = P_0 + t_0*(P_1-P_0)\) \(D_0 \equiv (P_1-P_0)\)
- \(E_2 = P_2 + t_2*(P_3-P_2)\) \(D_2 \equiv (P_3-P_2)\)
- \(P_0 + t_0*D_0 = P_2 + t_2*D_2\)
- \(x_0 + dx_0 * t_0 = x_2 + dx_2 * t_2\)
- \(y_0 + dy_0 * t_0 = y_2 + dy_2 * t_2\)
Intersecting Two Edges (2)

• Solve for t’s
  • $t_0 = \frac{((x_0 - x_2) * dy_2 + (y_2 - y_0) * dx_2)}{(dy_0 * dx_2 - dx_0 * dy_2)}$
  • $t_2 = \frac{((x_2 - x_0) * dy_0 + (y_0 - y_2) * dx_0)}{(dy_2 * dx_0 - dx_2 * dy_0)}$

• See http://www.vb-helper.com/howto_intersect_lines.html for derivation

• Edges intersect if $0 \leq t_0, t_2 \leq 1$
• Edges are parallel if denominator = 0
Examples

\[ 0 \leq t_0, t_2 \leq 1 \]

\[ t_0, t_2 \leq 0 \]

\[ t_2 \leq 0 \]
\[ 0 \leq t_0 \leq 1 \]
Filling Primitives: Rectangles, Polygons & Circles

• Two part process
  – Which pixels to fill?
  – What values to fill them with?

• Idea: **Coherence**
  – *Spatial*: pixels are the same from pixel-to-pixel and scan-line to scan line;
  – *Span*: all pixels on a span get the same value
  – *Scan-line*: consecutive scan lines are the same
  – *Edge*: pixels are the same along edges
Scan Filling Primitives: Rectangles

• Easy algorithm
  – Fill from $x_{\text{min}}$ to $x_{\text{max}}$
    Fill from $y_{\text{min}}$ to $y_{\text{max}}$

• Issues
  – What if two adjacent rectangles share an edge?
  – Color the boundary pixels twice?
  – Rules:
    • Color only interior pixels
    • Color left and bottom edges
Scan Filling Primitives: Polygons

• Observe:
  – FA, DC intersections are integer
  – FE, ED intersections are not integer

• For each scan line, how to figure out which pixels are inside the polygon?
Scan Filling Polygons

- Idea #1: use midpoint algo on each edge, fill in between extrema points
- Note: many extrema pixels lie outside the polygon
- Why: midpoint algo has no sense of in/out
Scan Filling Polygons

- **Idea #2**: draw pixels only strictly inside
  - Find intersections of scan line with edges
  - Sort intersections by increasing $x$ coordinate
  - Fill pixels on inside based on a parity bit
    - $B_p$ initially even (off)
    - Invert at each intersect
    - Draw when odd, do not draw when even
Scan Filling Polygons

• Issues with Idea #2:
  – If at a fractional x value, how to pick which pixels are in interior?
  – Intersections at integer vertex coordinates?
  – Shared vertices?
  – Vertices that define a horizontal edge?
How to handle vertices?

• Problem:
  – vertices are counted twice

• Solution:
  – If both neighboring vertices are on the same side of the scan line, don’t count it
  – If both neighboring vertices are on different sides of a scan line, count it once
  – Compare current y value with y value of neighboring vertices
Scan-Filling a Polygon
How to handle horizontal edges?

- Idea: don’t count their vertices
- Apply open and closed status to vertices to other edges
  - $y_{\text{min}}$ vertex closed
  - $y_{\text{max}}$ vertex is open
- On AB, A is at $y_{\text{min}}$ for JA; AB does not contribute, $B_p$ is odd and draw AB
- Edge BC has $y_{\text{min}}$ at B, but AB does not contribute, $B_p$ becomes even and drawing stops
How to handle horizontal edges?

- Start drawing at IJ ($B_p$ becomes odd).
- C is $y_{max}$ (open) for BC. $B_p$ doesn’t change.
- Ignore CD. D is $y_{min}$ (closed) for DE. $B_p$ becomes even. Stop drawing.
- I is $y_{max}$ (open) for IJ. No drawing.
- Ignore IH. H is $y_{min}$ (closed) for GH. $B_p$ becomes odd. Draw to FE.
- Ignore GF. No drawing
Polygon Filling Algorithm

- For each polygon
  - For each edge, mark each scan-line that the edge crosses by examining its $y_{min}$ and $y_{max}$
    - If edge is horizontal, ignore it
    - If $y_{max}$ on scan-line, ignore it
    - If $y_{min} \leq y < y_{max}$ add edge to scan-line $y$‘s edge list
  - For each scan-line between polygon’s $y_{min}$ and $y_{max}$
    - Calculate intersections with edges on list
    - Sort intersections in $x$
    - Perform parity-bit scan-line filling
    - Apply ceiling on first xsect and floor on second xsect
    - Check for double intersection special case
  - Clear scan-lines’ edge list
Example

Diagram with points E1, E2, E3, E4, E5, and E6.
How to handle slivers?

- When the scan area does not have an “interior”
- Solution: use anti-aliasing
- But, to do so will require softening the rules about drawing only interior pixels
Scan Filling Curved Objects

- Hard in general case
- Easier for circles and ellipses.
- Use midpoint Alg to generate boundary points.
- Fill in horizontal pixel spans
- Use symmetry
Boundary-Fill Algorithm

1. Start with some internal point \((x,y)\)
2. Color it
3. Check neighbors for filled or border color
4. Color neighbors if OK
5. Continue recursively
4 Connected Boundary-Fill Alg

Void BoundaryFill4( int x, int y, int fill, int bnd)
{
    If Color(x,y) != fill and Color(x,y) != bnd
    {
        SetColor(x,y) = fill;
        BoundaryFill4(x+1, y, fill, bnd);
        BoundaryFill4(x, y +1, fill, bnd);
        BoundaryFill4(x-1, y, fill, bnd);
        BoundaryFill4(x, y -1, fill, bnd);
    }
}
Boundary-Fill Algorithm

• Issues with recursive boundary-fill algorithm:
  – May make mistakes if parts of the space already filled with the Fill color
  – Requires very big stack size

• More efficient algorithms
  – First color contiguous span along one scan line
  – Only stack beginning positions of neighboring scan lines
Plain PBM Image files

- There is exactly one image in a file.
- The "magic number" is "P1" instead of "P4".
- Each pixel in the raster is represented by a byte containing ASCII '1' or '0', representing black and white respectively. There are no fill bits at the end of a row.
- White space in the raster section is ignored.
- You can put any junk you want after the raster, if it starts with a white space character.
- No line should be longer than 70 characters.

Here is an example of a small image in the plain PBM format.

```
P1
# feep.pbm
24 7
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 1 1 1 1 0 0 1 1 1 1 0 0 1 1 1 1 0 0 1 1 1 1 0
0 1 0 0 0 0 0 1 0 0 0 0 0 1 0 0 0 0 0 1 0 0 1 0
0 1 1 1 1 0 0 0 1 1 1 1 0 0 1 1 1 0 0 1 1 1 1 0
0 1 0 0 0 0 0 1 0 0 0 0 0 1 0 0 0 0 0 1 0 0 0 0
0 1 0 0 0 0 0 1 1 1 1 0 0 1 1 1 1 0 0 1 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
```

There is a newline character at the end of each of these lines.
Plain PBM Image File

- There is exactly one image in a file
- File begins with "magic number" "P1"
- Next line specifies pixel resolution
- Each pixel is represented by a byte containing ASCII ‘1’ (black) or ‘0’ (white)
- All fields/values separated by whitespace characters
- No line longer than 70 characters?
Plain PBM Image Example

P1
# feep.pbm
24 7
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 1 1 1 1 0 0 1 1 1 1 0 0 1 1 1 1 1 1 0 0 1 1 1 1 0
0 1 0 0 0 0 0 1 0 0 0 0 0 1 0 0 0 0 0 1 0 0 1 0
0 1 1 1 0 0 0 1 1 1 0 0 0 1 1 1 1 1 0 0 1 1 1 1 0
0 1 0 0 0 0 0 1 0 0 0 0 0 1 0 0 0 0 0 1 0 0 0 0
0 1 0 0 0 0 0 1 1 1 1 0 0 1 1 1 1 1 0 0 1 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

There is a newline character at the end of each of these lines.
Course Status

So far everything straight lines!

• How to model 2D curved objects?
  – Representation
    • Circles
    • Types of 2D Curves
    • Parametric Cubic Curves
    • Bézier Curves, (non)uniform, (non)rational
    • NURBS
  – Drawing of 2D Curves
    • Line drawing algorithms for complex curves
    • DeCasteljeau, Subdivision, De Boor
Homework #2

• Modify homework #1
• Add reading “moveto” and “lineto” commands
• They define closed polygons
• Transform polygon vertices
• Clip polygons against window with Sutherland-Hodgman algorithm
• Display edges with HW1 line-drawing code
Programming assignment 3

- Input PostScript-like file.
- Output B/W PBM.
- Implement viewports.
- Use HW2 for polygon clipping.
- Implement scanline polygon filling. (You can not use flood filling algorithms)