CS 430
Computer Graphics

Polygon Clipping and Filling
Week 3, Lecture 5

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Outline

• Polygon clipping
  – Sutherland-Hodgman,
  – Weiler-Atherton

• Polygon filling
  – Scan filling polygons
  – Flood filling polygons

• Introduction and discussion of homework #2
Polygon

- Ordered set of vertices (points)
  - Usually counter-clockwise
- Two consecutive vertices define an edge
- Left side of edge is inside
- Right side is outside
- Last vertex implicitly connected to first
- In 3D vertices should be co-planar
Polygon Clipping

• Lots of different cases
• Issues
  – Edges of polygon need to be tested against clipping rectangle
  – May need to add new edges
  – Edges discarded or divided
  – Multiple polygons can result from a single polygon
The Sutherland-Hodgman Polygon-Clipping Algorithm

• Divide and Conquer

• Idea:
  – Clip single polygon using single infinite clip edge
  – Repeat 4 times

• Note the generality:
  – 2D convex n-gons can clip arbitrary n-gons
  – 3D convex polyhedra can clip arbitrary polyhedra
Sutherland-Hodgman Algorithm

• Input:
  – $v_1, v_2, \ldots, v_n$ the vertices defining the polygon
  – Single infinite clip edge w/ inside/outside info

• Output:
  – $v'_1, v'_2, \ldots, v'_m$, vertices of the clipped polygon

• Do this 4 (or $n_e$) times

• Traverse vertices (edges)
• Add vertices one-at-a-time to output polygon
  – Use inside/outside info
  – Edge intersections
Sutherland-Hodgman Algorithm

- Can be done incrementally
- If first point inside add. If outside, don’t add
- Move around polygon from $v_1$ to $v_n$ and back to $v_1$
- Check $v_i, v_{i+1}$ wrt the clip edge
- Need $v_i, v_{i+1}$‘s inside/outside status
- Add vertex one at a time. There are 4 cases:

Case 1: Inside | Outside
---|---
Polygon being clipped | Clip boundary
$p$: output

Case 2: Inside | Outside
---|---
$p$: output

Case 3: Inside | Outside
---|---
(p: second output

Case 4: Inside | Outside
---|---
i: first output

1994 Foley/VanDam/Finer/Huges/Phillips ICG
Sutherland-Hodgman Algorithm

• Given polygon $P \quad P' = P$
  – foreach clipping edge (there are 4) {
    • Clip polygon $P'$ to clipping edge
      – foreach edge in polygon $P'$
        » Check clipping cases (there are 4)
          » Case 1 : Output $v_{i+1}$ to $P''$
          » Case 2 : Output intersection point to $P''$
          » Case 3 : No output
          » Case 4 : Output intersection point & $v_{i+1}$ to $P''$
    • $P' = P''$
  }

}
Sutherland-Hodgman Algorithm

Animated by Max Peysakhov @ Drexel University
Final Result

Note: Edges XY and ZW!
Issues with Sutherland-Hodgman Algorithm

• Clipping a concave polygon
• Can produce two CONNECTED areas
Weiler-Atherton Algorithm

- General clipping algorithm for concave polygons with holes
- Produces multiple polygons (with holes)
- Make linked list data structure
- Traverse to make new polygon(s)
Weiler-Atherton Algorithm

• Given polygons A and B as linked list of vertices (counter-clockwise order)
• Find all edge intersections & place in list
• Insert as “intersection” nodes
• Nodes point to A & B
• Determine in/out status of vertices
Linked List Data Structure

Intersection Nodes
Intersection Special Cases

- If “intersecting” edges are parallel, ignore
- Intersection point is a vertex
  - Vertex of A lies on a vertex or edge of B
  - Edge of A runs through a vertex of B
  - Replace vertex with an intersection node
Weiler-Atherton Algorithm: Union

• Find a vertex of A outside of B
• Traverse linked list
• At each intersection point switch to other polygon
• Do until return to starting vertex
• All visited vertices and nodes define union’ed polygon
Example: Union

\{V1, V2, V3, P0, V8, V4, P3, V0\}, \{V6, P1, P2\}
Example
Result
Weiler-Atherton Algorithm: Intersection

• Start at intersection point
  – If connected to an “inside” vertex, go there
  – Else step to an intersection point
  – If neither, stop
• Traverse linked list
• At each intersection point switch to other polygon and remove intersection point list
• Do until return to starting intersection point
• If intersection list not empty, pick another one
• All visited vertices and nodes define and’ed polygon
Example: Intersection

{P1, V7, P0}, {P3, V5, P2}
Boolean Special Cases

If polygons don’t intersect

– Union
  
  • If one inside the other, return polygon that surrounds the other
  
  • Else, return both polygons

– Intersection

  • If one inside the other, return polygon inside the other
  
  • Else, return no polygons
Point P Inside a Polygon?

- Connect P with another point P' that you know is outside polygon
- Intersect segment PP' with polygon edges
- Watch out for vertices!
- If # intersections is even (or 0) $\rightarrow$ Outside
- If odd $\rightarrow$ Inside
Point P Inside a Rectangle?

• Just re-use code from Cohen-Sutherland algorithm
• If a vertex’s bit code equals zero, it’s inside
• Else, it’s outside
Edge clipping

• Re-use line clipping from HW1
  – Similar triangles method
  – Cyrus-Beck line clipping

• Yet another technique
Intersecting Two Edges (1)

• Edge 0 : \((P_0, P_1)\)
• Edge 2 : \((P_2, P_3)\)
• \(E_0 = P_0 + t_0*(P_1-P_0)\) \(D_0 \equiv (P_1-P_0)\)
• \(E_2 = P_2 + t_2*(P_3-P_2)\) \(D_2 \equiv (P_3-P_2)\)
• \(P_0 + t_0*D_0 = P_2 + t_2*D_2\)
• \(x_0 + dx_0 * t_0 = x_2 + dx_2 * t_2\)
• \(y_0 + dy_0 * t_0 = y_2 + dy_2 * t_2\)
Intersecting Two Edges (2)

- Solve for t’s
- \( t_0 = \frac{((x_0 - x_2) \cdot dy_2 + (y_2 - y_0) \cdot dx_2)}{(dy_0 \cdot dx_2 - dx_0 \cdot dy_2)} \)
- \( t_2 = \frac{((x_2 - x_0) \cdot dy_0 + (y_0 - y_2) \cdot dx_0)}{(dy_2 \cdot dx_0 - dx_2 \cdot dy_0)} \)
- See [http://www.vb-helper.com/howto_intersect_lines.html](http://www.vb-helper.com/howto_intersect_lines.html) for derivation
- Edges intersect if \( 0 \leq t_0, t_2 \leq 1 \)
- Edges are parallel if denominator = 0
Examples

$0 \leq t_0, t_2 \leq 1$

t_0, t_2 \leq 0

t_2 \leq 0
\quad 0 \leq t_0 \leq 1
Filling Primitives: Rectangles, Polygons & Circles

• Two part process
  – Which pixels to fill?
  – What values to fill them with?

• Idea: Coherence
  – Spatial: pixels are the same from pixel-to-pixel and scan-line to scan line;
  – Span: all pixels on a span get the same value
  – Scan-line: consecutive scan lines are the same
  – Edge: pixels are the same along edges
Scan Filling Primitives: Rectangles

- Easy algorithm
  - Fill from $x_{\text{min}}$ to $x_{\text{max}}$
  - Fill from $y_{\text{min}}$ to $y_{\text{max}}$

- Issues
  - What if two adjacent rectangles share an edge?
  - Color the boundary pixels twice?
- Rules:
  - Color only interior pixels
  - Color left and bottom edges
Scan Filling Primitives: Polygons

- Observe:
  - FA, DC intersections are integer
  - FE, ED intersections are not integer

- For each scan line, how to figure out which pixels are inside the polygon?
Scan Filling Polygons

- Idea #1: use midpoint algo on each edge, fill in between extrema points
- Note: many extrema pixels lie outside the polygon
- Why: midpoint algo has no sense of in/out

(a) Span extrema    Other pixels in the span
Scan Filling Polygons

• Idea #2: draw pixels only strictly inside
  – Find intersections of scan line with edges
  – Sort intersections by increasing x coordinate
  – Fill pixels on inside based on a parity bit
    • $B_p$ initially even (off)
    • Invert at each intersect
    • Draw when odd, do not draw when even
Scan Filling Polygons

• Issues with Idea #2:
  – If at a fractional x value, how to pick which pixels are in interior?
  – Intersections at integer vertex coordinates?
  – Shared vertices?
  – Vertices that define a horizontal edge?
How to handle vertices?

- **Problem:**
  - vertices are counted twice

- **Solution:**
  - If both neighboring vertices are on the same side of the scan line, don’t count it
  - If both neighboring vertices are on different sides of a scan line, count it once
  - Compare current y value with y value of neighboring vertices
Scan-Filling a Polygon

Animated by Max Peysakhov @ Drexel University
How to handle horizontal edges?

- Idea: don’t count their vertices
- Apply open and closed status to vertices to other edges
  - $y_{\text{min}}$ vertex closed
  - $y_{\text{max}}$ vertex is open
- On AB, A is at $y_{\text{min}}$ for JA; AB does not contribute, $B_p$ is odd and draw AB
- Edge BC has $y_{\text{min}}$ at B, but AB does not contribute, $B_p$ becomes even and drawing stops
How to handle horizontal edges?

• Start drawing at IJ ($B_p$ becomes odd).
• C is $y_{max}$ (open) for BC. $B_p$ doesn’t change.
• Ignore CD. D is $y_{min}$ (closed) for DE. $B_p$ becomes even. Stop drawing.
• I is $y_{max}$ (open) for IJ. No drawing.
• Ignore IH. H is $y_{min}$ (closed) for GH. $B_p$ becomes odd. Draw to FE.
• Ignore GF. No drawing
Polygon Filling Algorithm

- For each polygon
  - For each edge, mark each scan-line $y$ that the edge crosses by examining its $y_{min}$ and $y_{max}$
    - If edge is horizontal, ignore it
    - If $y_{max}$ on scan-line, ignore it
    - If $y_{min} \leq y < y_{max}$ add edge to scan-line $y$'s edge list
  - For each scan-line between polygon’s $y_{min}$ and $y_{max}$
    - Calculate intersections with edges on list
    - Sort intersections in $x$
    - Perform parity-bit scan-line filling
    - Apply floor on first xsect and ceiling on second xsect
    - Check for double intersection special case
- Clear scan-lines’ edge list
Example

E1, E2, E3, E4, E5, E6
How to handle slivers?

• When the scan area does not have an “interior”
• Solution: use anti-aliasing
• But, to do so will require softening the rules about drawing only interior pixels
Scan Filling Curved Objects

- Hard in general case
- Easier for circles and ellipses.
- Use midpoint Alg to generate boundary points.
- Fill in horizontal pixel spans
- Use symmetry
Boundary-Fill Algorithm

- Start with some internal point \((x,y)\)
- Color it
- Check neighbors for filled or border color
- Color neighbors if OK
- Continue recursively
Void BoundaryFill4( int x, int y, int fill, int bnd)
{
    If Color(x,y) != fill and Color(x,y) != bnd
    {
        SetColor(x,y) = fill;
        BoundaryFill4(x+1, y, fill, bnd);
        BoundaryFill4(x, y+1, fill, bnd);
        BoundaryFill4(x-1, y, fill, bnd);
        BoundaryFill4(x, y-1, fill, bnd);
    }
}
Boundary-Fill Algorithm

• Issues with recursive boundary-fill algorithm:
  – May make mistakes if parts of the space already filled with the Fill color
  – Requires very big stack size

• More efficient algorithms
  – First color contiguous span along one scan line
  – Only stack beginning positions of neighboring scan lines
Plain PBM Image files

- There is exactly one image in a file.
- The "magic number" is "P1" instead of "P4".
- Each pixel in the raster is represented by a byte containing ASCII '1' or '0', representing black and white respectively. There are no fill bits at the end of a row.
- White space in the raster section is ignored.
- You can put any junk you want after the raster, if it starts with a white space character.
- No line should be longer than 70 characters.

Here is an example of a small image in the plain PBM format.

```
P1
# feep.pbm
24 7
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 1 1 1 1 0 0 1 1 1 1 0 0 1 1 1 1 0 0 1 1 1 1 0
0 1 0 0 0 0 0 1 0 0 0 0 0 1 0 0 0 0 0 1 0 0 1 0
0 1 1 1 0 0 0 1 1 1 1 0 0 1 1 1 1 0 0 1 1 1 1 0
0 1 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0 1 0 0 1 0
0 1 0 0 0 0 0 1 1 1 1 1 0 0 1 1 1 1 1 0 0 1 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
```

There is a newline character at the end of each of these lines.
Plain PBM Image File

- There is exactly one image in a file
- File begins with ”magic number” “P1”
- Next line specifies pixel resolution
- Each pixel is represented by a byte containing ASCII ‘1’ (black) or ‘0’ (white)
- All fields/values separated by whitespace characters
- No line longer than 70 characters?
Plain PBM Image Example

P1
# feep.pbm
24 7
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 1 1 1 1 0 0 1 1 1 1 0 0 1 1 1 1 0 0 1 1 1 1 0
0 1 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 1 0 0 0 0 0 0 1 0 0 1 0
0 1 1 1 0 0 0 1 1 1 0 0 0 1 1 1 0 0 0 1 1 1 1 0
0 1 0 0 0 0 0 0 1 0 0 0 0 0 1 0 0 0 0 0 1 0 0 0 0
0 1 0 0 0 0 0 0 1 1 1 1 0 0 1 1 1 1 0 0 1 1 1 1 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

There is a newline character at the end of each of these lines.
Course Status

So far everything straight lines!

• How to model 2D curved objects?
  – Representation
    • Circles
    • Types of 2D Curves
    • Parametric Cubic Curves
    • Bézier Curves, (non)uniform, (non)rational
    • NURBS
  – Drawing of 2D Curves
    • Line drawing algorithms for complex curves
    • DeCasteljeau, Subdivision, De Boor
Homework #2

- Modify homework #1
- Add reading “moveto” and “lineto” commands
- They define closed polygons
- Transform polygon vertices
- Clip polygons against window with Sutherland-Hodgman algorithm
- Display edges with HW1 line-drawing code
Programming assignment 3

- Input PostScript-like file.
- Output B/W PBM.
- Implement viewports.
- Use HW2 for polygon clipping.
- Implement scanline polygon filling. (*You can not use flood filling algorithms*)