CS 430
Computer Graphics

Polygon Clipping and Filling
Week 3, Lecture 5

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Outline

• Polygon clipping
  – Sutherland-Hodgman,
  – Weiler-Atherton

• Polygon filling
  – Scan filling polygons
  – Flood filling polygons

• Introduction and discussion of homework #2
Polygon

- Ordered set of vertices (points)
  - Usually counter-clockwise
- Two consecutive vertices define an edge
- Left side of edge is inside
- Right side is outside
- Last vertex implicitly connected to first
- In 3D vertices should be co-planar
Polygon Clipping

• Lots of different cases

• Issues
  – Edges of polygon need to be tested against clipping rectangle
  – May need to add new edges
  – Edges discarded or divided
  – Multiple polygons can result from a single polygon
The Sutherland-Hodgman Polygon-Clipping Algorithm

• Divide and Conquer
• Idea:
  – Clip single polygon using single infinite clip edge
  – Repeat 4 times
• Note the generality:
  – 2D convex n-gons can clip arbitrary n-gons
  – 3D convex polyhedra can clip arbitrary polyhedra
Sutherland-Hodgman Algorithm

• Input:
  – \( v_1, v_2, \ldots, v_n \) the vertices defining the polygon
  – Single infinite clip edge w/ inside/outside info

• Output:
  – \( v'_1, v'_2, \ldots, v'_m \), vertices of the clipped polygon

• Do this 4 (or \( n_e \)) times

• Traverse vertices (edges)

• Add vertices one-at-a-time to output polygon
  – Use inside/outside info
  – Edge intersections
Sutherland-Hodgman Algorithm

- Can be done incrementally
- If first point inside add. If outside, don’t add
- Move around polygon from \( v_1 \) to \( v_n \) and back to \( v_1 \)
- Check \( v_i, v_{i+1} \) wrt the clip edge
- Need \( v_i, v_{i+1} \)‘s inside/outside status
- Add vertex one at a time. There are 4 cases:

**Case 1**
- Inside
- Outside

**Case 2**
- Inside
- Outside

**Case 3**
- Inside
- Outside

**Case 4**
- Inside
- Outside

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Sutherland-Hodgman Algorithm

• Given polygon \( P \quad P' = P \)
  – foreach clipping edge (there are 4) {
    • Clip polygon \( P' \) to clipping edge
      – foreach edge in polygon \( P' \)
        » Check clipping cases (there are 4)
          » Case 1: Output \( v_{i+1} \) to \( P'' \)
          » Case 2: Output intersection point to \( P'' \)
          » Case 3: No output
          » Case 4: Output intersection point \& \( v_{i+1} \) to \( P'' \)
    • \( P' = P'' \)
  }

Sutherland-Hodgman Algorithm

Animated by Max Peysakhov @ Drexel University
Final Result

\{A, B, X, Y, E, Z, W, A\}

Note: Edges XY and ZW!
Issues with Sutherland-Hodgman Algorithm

- Clipping a concave polygon
- Can produce two CONNECTED areas
Weiler-Atherton Algorithm

• General clipping algorithm for concave polygons with holes
• Produces multiple polygons (with holes)
• Make linked list data structure
• Traverse to make new polygon(s)
Weiler-Atherton Algorithm

• Given polygons A and B as linked list of vertices (counter-clockwise order)
• Find all edge intersections & place in list
• Insert as “intersection” nodes
• Nodes point to A & B
• Determine in/out status of vertices
Linked List Data Structure

Intersection Nodes
Intersection Special Cases

• If “intersecting” edges are parallel, ignore
• Intersection point is a vertex
  – Vertex of A lies on a vertex or edge of B
  – Edge of A runs through a vertex of B
  – Replace vertex with an intersection node
Weiler-Atherton Algorithm: Union

• Find a vertex of A outside of B
• Traverse linked list
• At each intersection point switch to other polygon
• Do until return to starting vertex
• All visited vertices and nodes define union’ed polygon
Example: Union

\{V1, V2, V3, P0, V8, V4, P3, V0\}, \{V6, P1, P2\}
Example
Result
Weiler-Atherton Algorithm: Intersection

- Start at intersection point
  - If connected to an “inside” vertex, go there
  - Else step to an intersection point
  - If neither, stop
- Traverse linked list
- At each intersection point switch to other polygon and remove intersection point list
- Do until return to starting intersection point
- If intersection list not empty, pick another one
- All visited vertices and nodes define and’ed polygon
Example: Intersection

{P1, V7, P0}, {P3, V5, P2}
Boolean Special Cases

If polygons don’t intersect

– Union
  • If one inside the other, return polygon that surrounds the other
  • Else, return both polygons

– Intersection
  • If one inside the other, return polygon inside the other
  • Else, return no polygons
Point P Inside a Polygon?

- Connect P with another point P` that you know is outside polygon
- Intersect segment PP` with polygon edges
- Watch out for vertices!
- If # intersections is even (or 0) → Outside
- If odd → Inside
Point P Inside a Rectangle?

- Just re-use code from Cohen-Sutherland algorithm
- If a vertex’s code equals zero, it’s inside
- Else, it’s outside
Edge clipping

- Re-use line clipping from HW1
  - Similar triangles method
  - Cyrus-Beck line clipping
- Yet another technique
Intersecting Two Edges (1)

• Edge 0 : \((P_0, P_1)\)
• Edge 2 : \((P_2, P_3)\)
• \(E_0 = P_0 + t_0*(P_1-P_0)\) \quad D_0 \equiv (P_1-P_0)
• \(E_2 = P_2 + t_2*(P_3-P_2)\) \quad D_2 \equiv (P_3-P_2)
• \(P_0 + t_0*D_0 = P_2 + t_2*D_2\)
• \(x_0 + dx_0 \times t_0 = x_2 + dx_2 \times t_2\)
• \(y_0 + dy_0 \times t_0 = y_2 + dy_2 \times t_2\)
Intersecting Two Edges (2)

• Solve for t’s
• $t_0 = \frac{((x_0 - x_2) \cdot dy_2 + (y_2 - y_0) \cdot dx_2)}{(dy_0 \cdot dx_2 - dx_0 \cdot dy_2)}$
• $t_2 = \frac{((x_2 - x_0) \cdot dy_0 + (y_0 - y_2) \cdot dx_0)}{(dy_2 \cdot dx_0 - dx_2 \cdot dy_0)}$

• See http://www.vb-helper.com/howto_intersect_lines.html for derivation
• Edges intersect if $0 \leq t_0, t_2 \leq 1$
• Edges are parallel if denominator = 0
Examples

\[0 \leq t_0, t_2 \leq 1\]

\[t_0, t_2 \leq 0\]

\[t_2 \leq 0, \quad 0 \leq t_0 \leq 1\]
Filling Primitives: Rectangles, Polygons & Circles

• Two part process
  – Which pixels to fill?
  – What values to fill them with?

• Idea: **Coherence**
  – *Spatial*: pixels are the same from pixel-to-pixel and scan-line to scan line;
  – *Span*: all pixels on a span get the same value
  – *Scan-line*: consecutive scan lines are the same
  – *Edge*: pixels are the same along edges
Scan Filling Primitives: Rectangles

• Easy algorithm
  – Fill from $x_{\text{min}}$ to $x_{\text{max}}$
  – Fill from $y_{\text{min}}$ to $y_{\text{max}}$

• Issues
  – What if two adjacent rectangles share an edge?
  – Color the boundary pixels twice?
  – Rules:
    • Color only interior pixels
    • Color left and bottom edges
Scan Filling Primitives: Polygons

- Observe:
  - FA, DC intersections are integer
  - FE, ED intersections are not integer
- For each scan line, how to figure out which pixels are inside the polygon?
Scan Filling Polygons

- Idea #1: use midpoint algo on each edge, fill in between extrema points
- Note: many extrema pixels lie outside the polygon
- Why: midpoint algo has no sense of in/out

(a)

- Span extrema
- Other pixels in the span
Scan Filling Polygons

- Idea #2: draw pixels only strictly inside
  - Find intersections of scan line with edges
  - Sort intersections by increasing x coordinate
  - Fill pixels on inside based on a parity bit
    - $B_p$ initially even (off)
    - Invert at each intersect
    - Draw when odd, do not draw when even

● Span extrema  ● Other pixels in the span

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Scan Filling Polygons

• Issues with Idea #2:
  – If at a fractional x value, how to pick which pixels are in interior?
  – Intersections at integer vertex coordinates?
  – Shared vertices?
  – Vertices that define a horizontal edge?
How to handle vertices?

• Problem:
  – vertices are counted twice
• Solution:
  – If both neighboring vertices are on the same side of the scan line, don’t count it
  – If both neighboring vertices are on different sides of a scan line, count it once
  – Compare current y value with y value of neighboring vertices
Scan-Filling a Polygon
How to handle horizontal edges?

- Idea: don’t count their vertices
- Apply open and closed status to vertices to other edges
  - $y_{\text{min}}$ vertex closed
  - $y_{\text{max}}$ vertex is open
- On AB, A is at $y_{\text{min}}$ for JA; AB does not contribute, $B_p$ is odd and draw AB
- Edge BC has $y_{\text{min}}$ at B, but AB does not contribute, $B_p$ becomes even and drawing stops
How to handle horizontal edges?

- Start drawing at IJ ($B_p$ becomes odd).
- C is $y_{max}$ (open) for BC. $B_p$ doesn’t change.
- Ignore CD. D is $y_{min}$ (closed) for DE. $B_p$ becomes even. Stop drawing.
- I is $y_{max}$ (open) for IJ. No drawing.
- Ignore IH. H is $y_{min}$ (closed) for GH. $B_p$ becomes odd. Draw to FE.
- Ignore GF. No drawing.
Polygon Filling Algorithm

• For each polygon
  – For each edge, mark each scan-line that the edge crosses by examining its $y_{min}$ and $y_{max}$
    • If edge is horizontal, ignore it
    • If $y_{max}$ on scan-line, ignore it
    • If $y_{min} \leq y < y_{max}$ add edge to scan-line $y$‘s edge list
  – For each scan-line between polygon’s $y_{min}$ and $y_{max}$
    • Calculate intersections with edges on list
    • Sort intersections in $x$
    • Perform parity-bit scan-line filling
    • Check for double intersection special case
  – Clear scan-lines’ edge list
Example
How to handle slivers?

- When the scan area does not have an “interior”
- Solution: use anti-aliasing
- But, to do so will require softening the rules about drawing only interior pixels
Scan Filling Curved Objects

- Hard in general case
- Easier for circles and ellipses.
- Use midpoint Alg to generate boundary points.
- Fill in horizontal pixel spans
- Use symmetry
Boundary-Fill Algorithm

- Start with some internal point \((x,y)\)
- Color it
- Check neighbors for filled or border color
- Color neighbors if OK
- Continue recursively
4 Connected Boundary-Fill Alg

Void BoundaryFill4( int x, int y, int fill, int bnd)
{
    If Color(x,y) != fill and Color(x,y) != bnd
    {
        SetColor(x,y) = fill;
        BoundaryFill4(x+1, y, fill, bnd);
        BoundaryFill4(x, y +1, fill, bnd);
        BoundaryFill4(x-1, y, fill, bnd);
        BoundaryFill4(x, y -1, fill, bnd);
    }
}
Boundary-Fill Algorithm

• Issues with recursive boundary-fill algorithm:
  – May make mistakes if parts of the space already filled with the Fill color
  – Requires very big stack size

• More efficient algorithms
  – First color contiguous span along one scan line
  – Only stack beginning positions of neighboring scan lines

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Course Status

So far everything straight lines!

• How to model 2D curved objects?
  – Representation
    • Circles
    • Types of 2D Curves
    • Parametric Cubic Curves
    • Bézier Curves, (non)uniform, (non)rational
    • NURBS
  – Drawing of 2D Curves
    • Line drawing algorithms for complex curves
    • DeCasteljeau, Subdivision, De Boor
Homework #2

- Modify homework #1
- Add reading “moveto” and “lineto” commands
- They define closed polygons
- Transform polygon vertices
- Clip polygons against window with Sutherland-Hodgman algorithm
- Display edges with HW1 line-drawing code
Programming assignment 3

• Input PostScript-like file.
• Output B/W PBM.
• Implement viewports.
• Use HW2 for polygon clipping.
• Implement scanline polygon filling. (*You can not use flood filling algorithms*)