Polygon Clipping and Filling

Week 3, Lecture 5

David Breen, William Regli and Maxim Peysakhov
Department of Computer Science
Drexel University
Outline

• Polygon clipping
  – Sutherland-Hodgman,
  – Weiler-Atherton

• Polygon filling
  – Scan filling polygons
  – Flood filling polygons

• Introduction and discussion of homework #2
Polygon

- Ordered set of vertices (points)
  - Usually counter-clockwise
- Two consecutive vertices define an edge
- Left side of edge is inside
- Right side is outside
- Last vertex implicitly connected to first
- In 3D vertices should be co-planar
Polygon Clipping

• Lots of different cases
• Issues
  – Edges of polygon need to be tested against clipping rectangle
  – May need to add new edges
  – Edges discarded or divided
  – Multiple polygons can result from a single polygon
The Sutherland-Hodgman Polygon-Clipping Algorithm

• Divide and Conquer

• Idea:
  – Clip single polygon using single infinite clip edge
  – Repeat 4 times

• Note the generality:
  – 2D convex n-gons can clip arbitrary n-gons
  – 3D convex polyhedra can clip arbitrary polyhedra
Sutherland-Hodgman Algorithm

- **Input:**
  - $v_1, v_2, \ldots, v_n$ the vertices defining the polygon
  - Single infinite clip edge w/ inside/outside info

- **Output:**
  - $v'_1, v'_2, \ldots, v'_m$, vertices of the clipped polygon

- Do this 4 (or $n_e$) times

- **Traverse vertices (edges)**

- **Add vertices one-at-a-time to output polygon**
  - Use inside/outside info
  - Edge intersections
Sutherland-Hodgman Algorithm

- Can be done incrementally
- If first point inside add. If outside, don’t add
- Move around polygon from $v_1$ to $v_n$ and back to $v_1$
- Check $v_i, v_{i+1}$ wrt the clip edge
- Need $v_i, v_{i+1}$‘s inside/outside status
- Add vertex one at a time. There are 4 cases:
Sutherland-Hodgman Algorithm

• foreach polygon $P$  $P' = P$
  – foreach clipping edge (there are 4) {
    • Clip polygon $P'$ to clipping edge
      – foreach edge in polygon $P'$
        » Check clipping cases (there are 4)
          » Case 1 : Output $v_{i+1}$
          » Case 2 : Output intersection point
          » Case 3 : No output
          » Case 4 : Output intersection point $\& v_{i+1}$
Sutherland-Hodgman Algorithm

\{A, B, C, D, E, F, G, H, A\}
Sutherland-Hodgman Algorithm

Animated by Max Peysakhov @ Drexel University
Final Result

Note: Edges XY and ZW!
Issues with Sutherland-Hodgman Algorithm

- Clipping a concave polygon
- Can produce two CONNECTED areas
Weiler-Atherton Algorithm

- General clipping algorithm for concave polygons with holes
- Produces multiple polygons (with holes)
- Make linked list data structure
- Traverse to make new polygon(s)
Weiler-Atherton Algorithm

- Given polygons A and B as linked list of vertices (counter-clockwise order)
- Find all edge intersections & place in list
- Insert as “intersection” nodes
- Nodes point to A & B
- Determine in/out status of vertices
Linked List Data Structure

Intersection Nodes
Intersection Special Cases

- If “intersecting” edges are parallel, ignore
- Intersection point is a vertex
  - Vertex of A lies on a vertex or edge of B
  - Edge of A runs through a vertex of B
  - Replace vertex with an intersection node
Weiler-Atherton Algorithm: Union

- Find a vertex of A outside of B
- Traverse linked list
- At each intersection point switch to other polygon
- Do until return to starting vertex
- All visited vertices and nodes define union’ed polygon
Example: Union

\{V1, V2, V3, P0, V8, V4, P3, V0\},  \{V6, P1, P2\}
Weiler-Atherton Algorithm: Intersection

- Start at intersection point
  - If connected to an “inside” vertex, go there
  - Else step to an intersection point
  - If neither, stop
- Traverse linked list
- At each intersection point switch to other polygon and remove intersection point list
- Do until return to starting intersection point
- If intersection list not empty, pick another one
- All visited vertices and nodes define and’ed polygon
Example: Intersection

\{P1, V7, P0\}, \{P3, V5, P2\}
Boolean Special Cases

If polygons don’t intersect

- Union
  - If one inside the other, return polygon that surrounds the other
  - Else, return both polygons

- Intersection
  - If one inside the other, return polygon inside the other
  - Else, return no polygons
Point P Inside a Polygon?

- Connect P with another point P' that you know is outside polygon
- Intersect segment PP' with polygon edges
- Watch out for vertices!
- If # intersections is even (or 0) → Outside
- If odd → Inside
Edge clipping

• Re-use line clipping from HW1
  – Similar triangles method
  – Cyrus-Beck line clipping
• Yet another technique
Intersecting Two Edges (1)

- Edge 0 : \((P_0, P_1)\)
- Edge 2 : \((P_2, P_3)\)
- \(E_0 = P_0 + t_0 \cdot (P_1 - P_0)\) \quad \(D_0 = (P_1 - P_0)\)
- \(E_2 = P_2 + t_2 \cdot (P_3 - P_2)\) \quad \(D_2 = (P_3 - P_2)\)
- \(P_0 + t_0 \cdot D_0 = P_2 + t_2 \cdot D_2\)
- \(x_0 + dx_0 \cdot t_0 = x_2 + dx_2 \cdot t_2\)
- \(y_0 + dy_0 \cdot t_0 = y_2 + dy_2 \cdot t_2\)
Intersecting Two Edges (2)

- Solve for t’s
- \( t_0 = \frac{((x_0 - x_2) * dy_2 + (y_2 - y_0) * dx_2)}{(dy_0 * dx_2 - dx_0 * dy_2)} \)
- \( t_2 = \frac{((x_2 - x_0) * dy_0 + (y_0 - y_2) * dx_0)}{(dy_2 * dx_0 - dx_2 * dy_0)} \)
- See http://www.vb-helper.com/howto_intersect_lines.html for derivation
- Edges intersect if \( 0 \leq t_0, t_2 \leq 1 \)
- Edges are parallel if denominator = 0
Filling Primitives: Rectangles, Polygons & Circles

• Two part process
  – Which pixels to fill?
  – What values to fill them with?

• Idea: **Coherence**
  – *Spatial*: pixels are the same from pixel-to-pixel and scan-line to scan line;
  – *Span*: all pixels on a span get the same value
  – *Scan-line*: consecutive scan lines are the same
  – *Edge*: pixels are the same along edges
Scan Filling Primitives: Rectangles

• Easy algorithm
  – Fill from $x_{\text{min}}$ to $x_{\text{max}}$
  – Fill from $y_{\text{min}}$ to $y_{\text{max}}$

• Issues
  – What if two adjacent rectangles share an edge?
  – Color the boundary pixels twice?
  – Rules:
    • Color only interior pixels
    • Color left and bottom edges
Scan Filling Primitives: Polygons

• Observe:
  – FA, DC intersections are integer
  – FE, ED intersections are not integer

• For each scan line, how to figure out which pixels are inside the polygon?
Scan Filling Polygons

- Idea #1: use midpoint algo on each edge, fill in between extrema points
- Note: many extrema pixels lie outside the polygon
- Why: midpoint algo has no sense of in/out
Scan Filling Polygons

• Idea #2: draw pixels only strictly inside
  – Find intersections of scan line with edges
  – Sort intersections by increasing x coordinate
  – Fill pixels on inside based on a parity bit
    • $B_p$ initially even (off)
    • Invert at each intersect
    • Draw when odd, do not draw when even
Scan Filling Polygons

• Issues with Idea #2:
  – If at a fractional x value, how to pick which pixels are in interior?
  – Intersections at integer vertex coordinates?
  – Shared vertices?
  – Vertices that define a horizontal edge?
How to handle vertices?

- **Problem:**
  - vertices are counted twice

- **Solution:**
  - If both neighboring vertices are on the same side of the scan line, don’t count it
  - If both neighboring vertices are on different sides of a scan line, count it once
  - Compare current y value with y value of neighboring vertices
How to handle horizontal edges?

- Idea: don’t count their vertices
- Apply open and closed status to vertices to other edges
  - $y_{\text{min}}$ vertex closed
  - $y_{\text{max}}$ vertex is open
- On AB, A is at $y_{\text{min}}$ for JA; AB does not contribute, $B_p$ is odd and draw AB
- Edge BC has $y_{\text{min}}$ at B, but AB does not contribute, $B_p$ becomes even and drawing stops
How to handle horizontal edges?

- Start drawing at IJ \((B_p \text{ becomes odd})\).
- \(C\) is \(y_{\text{max}}\) (open) for BC. \(B_p\) doesn’t change.
- Ignore CD. D is \(y_{\text{min}}\) (closed) for DE. \(B_p\) becomes even. Stop drawing.
- I is \(y_{\text{max}}\) (open) for IJ. No drawing.
- Ignore IH. H is \(y_{\text{min}}\) (closed) for GH. \(B_p\) becomes odd. Draw to FE.
- Ignore GF. No drawing
Polygon Filling Algorithm

• For each polygon
  – For each edge, mark each scan-line that the edge crosses by examining its $y_{min}$ and $y_{max}$
    • If edge is horizontal, ignore it
    • If $y_{max}$ on scan-line, ignore it
    • If $y_{min} \leq y < y_{max}$ add edge to scan-line $y$‘s edge list
  – For each scan-line between polygon’s $y_{min}$ and $y_{max}$
    • Calculate intersections with edges on list
    • Sort intersections in $x$
    • Perform parity-bit scan-line filling
    • Check for double intersection special case
  – Clear scan-lines’ edge list
How to handle slivers?

- When the scan area does not have an “interior”
- Solution: use anti-aliasing
- But, to do so will require softening the rules about drawing only interior pixels
Scan-Filling a Polygon

Animated by Max Peysakhov @ Drexel University
Scan Filling Curved Objects

- Hard in general case
- Easier for circles and ellipses.
- Use midpoint Alg to generate boundary points.
- Fill in horizontal pixel spans
- Use symmetry
Boundary-Fill Algorithm

- Start with some internal point \((x,y)\)
- Color it
- Check neighbors for filled or border color
- Color neighbors if OK
- Continue recursively
4 Connected Boundary-Fill Alg

Void BoundaryFill4( int x, int y, int fill, int bnd)
{}
    If Color(x,y) != fill and Color(x,y) != bnd
    {
        SetColor(x,y) = fill;
        BoundaryFill4(x+1, y, fill, bnd);
        BoundaryFill4(x, y +1, fill, bnd);
        BoundaryFill4(x-1, y, fill, bnd);
        BoundaryFill4(x, y -1, fill, bnd);
    
    }
Boundary-Fill Algorithm

• Issues with recursive boundary-fill algorithm:
  – May make mistakes if parts of the space already filled with the Fill color
  – Requires very big stack size

• More efficient algorithms
  – First color contiguous span along one scan line
  – Only stack beginning positions of neighboring scan lines
Course Status

So far everything straight lines!

• How to model 2D curved objects?
  – Representation
    • Circles
    • Types of 2D Curves
    • Parametric Cubic Curves
    • Bézier Curves, (non)uniform, (non)rational
    • NURBS
  – Drawing of 2D Curves
    • Line drawing algorithms for complex curves
    • DeCasteljeau, Subdivision, De Boor
Homework #2

• Modify homework #1
• Add “moveto” and “lineto” commands
• They define closed polygons
• Clip polygons against window with Sutherland-Hodgman algorithm
• Display edges with HW1 line-drawing code