Recall the 2D Problem

- Objects exist in a 2D WCS
- Objects clipped/transformed to viewport
- Viewport transformed and drawn on 2D screen

From 3D Virtual World to 2D Screen

- Not unlike The Allegory of the Cave (Plato’s "Republic", Book VII)
- Viewers see a 2D shadow of 3D world
- How do we create this shadow?
- How do we make it as realistic as possible?

History of Linear Perspective

- Renaissance artists
  - Alberti (1435)
  - Della Francesca (1470)
  - Da Vinci (1490)
  - Pélérin (1505)
  - Dürer (1525)

The 3D Problem: Using a Synthetic Camera

- Think of 3D viewing as taking a photo:
  - Select Projection
  - Specify viewing parameters
  - Clip objects in 3D
  - Project the results onto the display and draw
The 3D Problem: (Slightly) Alternate Approach

- Think of 3D viewing as taking a photo:
  - Select Projection
  - Specify viewing parameters
  - Perform trivial accept/reject test in 3D
  - Project the results onto the image plane
  - Clip lines to world window
  - Transform to viewport and draw

Creating a 3D View: Parameterizing the Camera

Basic Ideas:
- Camera has
  - location
  - lens (focal length)
  - projection type
- World has
  - lights
  - colors
  - objects (visible and hidden surfaces)

Planar Geometric Projections

- Projections onto Planes
  - Consider the line AB
- Perspective Projection
  - a single viewing location
  - similar to a photograph
- Parallel Projection
  - viewing location at ∞
  - good for capturing shape and dimensions

Perspective Projections

- Idea: lines not parallel to projection plane converge to a vanishing point (VP)
- Lines extending to axis VPs are parallel to either x, y or z axes
- Projections characterized by # of axes cut by the projection plane

Perspective Projections: Example

- One-point perspective
- z axis vanishing point
- Projection plane cuts only the z axis

Perspective Projection (Titanic)
Perspective Projections:
Example
• Two-point perspective, cutting x and z
• Used commonly in CAD
• Three-point projections are not much different

Parallel Projections
• Two types, depending on projection direction vector and projection plane normal
  • Orthographic Projections
    – both vectors are the same
    – front-, top-, plan-, and side-elevation projections
  • Oblique Projections
    – vectors are different

Mercury Spacecraft

Axonometric Orthographic Projections
• Projections to planes not normal to principle coordinate axes, i.e. showing several faces
• The Isometric Projection
  – very common
  – projection plane at equal angles to each of the coordinate axes
  – 8 of them, one in each octant

Mercury Spacecraft

Oblique Projections
• Projection direction and Projection plane normal differ
• Preserves certain angles and distances
• Good for use in illustration and measurement
Oblique Projections

- Cavalier - all lines (including receding lines) are made to their true length
- Cabinet - receding lines are shortened by one-half their true length to approximate perspective foreshortening

Oblique Projections are Good for Illustrations

Projection Relationships

- As the distance to the projection point moves toward infinity, the two projection families unify
  - Projection plane
  - Direction to center of projection
  - Distance to CoP

Specification of 3D Views

- Projection Plane == View Plane
  - defined as a view reference point (VRP) and a view plane normal (VPN)
  - View up vector (VUP) defines “up” on the plane (so we can orient axes on to the plane)

Aiming the Projection

- Defined by:
  - Projection Reference Point (PRP)
  - Projection type
  - PRP is defined in with View Reference Coordinates (VRC)
  - Result: a semi-infinite viewing pyramid or view parallelepiped
- Perspective
  - CoP = PRP
- Parallel
  - DoP = CW - PRP
Defining the View Volume

• What portion of the world do we view?
  – where do we start?
  – how far back to go?
• View Volume
  – front clipping plane
  – back clipping plane
• For perspective, things far away gets smaller

From View Volume to Screen

• Consider a unit cube in normalized projection coordinates (NPC)
• Transform view volume to a rectangular solid in NPC
  – z-max plane: front clip plane
  – z-min plane: back clip plane
  – etc. for x and y
  – this is the 3D Viewport
• Transformation via the View Mapping Matrix
  – The z=1 face is mapped to the display
  – Display by discarding the z coordinate and drawing as in 2D

Parameter Summary

• Viewing Parameters:
  – VRP (WC)
  – VPN (WC)
  – VUP (WC)
  – PRP (VRC)
  – \( (u,v)_{\text{min}} \) \( (u,v)_{\text{max}} \)
  – CW (VRC)
  – F & B (VRC)
  – projection type
• What the parameters mean:
  – View Reference Point
  – View Plane Normal
  – View Up Vector
  – Projection Reference Point
  – Window extent
  – Center of Window
  – Front and Back clipping planes
  – perspective/parallel

Parameterizing Projections

• Viewing Parameters:
  – VRP (WC)
  – VPN (WC)
  – VUP (WC)
  – PRP (VRC)
  – window (VRC)
  – projection type
• What the parameters mean:
  – View Reference Point
  – View Plane Normal
  – View Up Vector
  – Projection Reference Point
  – Size of the 2D window
  – perspective/parallel

Examples of 3D Viewing: Preliminaries

• Dimensions and location of a simple house
• Two-point perspective projection of the house

Examples of 3D Viewing: Preliminaries

• Default viewing specification
  – \( x,y,z \) coincides with \( u,v,n \)
  – Window bounds from 0 to 1
Examples of 3D Viewing: Preliminaries

- Default parallel projection view volume
  - cuboidal

Examples of 3D Viewing: Preliminaries

- Default perspective projection view volume
  - pyramid-like

Parameterizing Projections: Example

- Viewing Parameters:
  - VRP(WC) (0,0,0)
  - VPN(WC) (0,0,1)
  - VUP(WC) (0,1,0)
  - PRP(VRC) (5.5,1.0)
  - window(VRC) (0,1,0,1)
  - projection parallel
  - DOP(VRC) (0,0,-1)

Perspective Projections: Example

- Parameters:
  - VRP(WC) (0,0,0)
  - VPN(WC) (0,0,1)
  - VUP(WC) (0,1,0)
  - PRP(VRC) (8,6,84)
  - window(VRC) (-50,50,-50,50)
  - projection perspective

Perspective Projections: Example (centering)

- Parameters:
  - VRP(WC) (0,0,54)
  - VPN(WC) (0,0,1)
  - VUP(WC) (0,1,0)
  - PRP(VRC) (8,6,30)
  - window(VRC) (-1.17,-1.17)
  - projection perspective

Perspective Projections: Example (centering 2)

- Parameters:
  - VRP(WC) (8,6,54)
  - VPN(WC) (0,0,1)
  - VUP(WC) (0,1,0)
  - PRP(VRC) (0,0,30)
  - window(VRC) (-8.9,-7.11)
  - projection perspective
Finite View Volumes: Example

- Parameters:
  - VRP(WC) = (0.0, 54)
  - VPN(WC) = (0, 1)
  - VUP(WC) = (0, 1.0)
  - PRP(VRC) = (8.6, 30)
  - window(VRC) = (-1.17, -1.17)
  - projection = perspective
  - B(VRC) = -23

Perspective Projections: Example

- Parameters:
  - VRP(WC) = (16, 0.54)
  - VPN(WC) = (0.0, 1)
  - VUP(WC) = (0.1, 0)
  - PRP(VRC) = (20.25, 20)
  - window(VRC) = (-20.20, -5.35)
  - projection = perspective

Perspective Projections: Example (cont.)

- Showing the object relative to the view plane, w/ overhead view

Perspective Projections: Example (rotating VUP)

- Same parameters as before
- VUP rotated away from y by 10°

Parallel Projections: Example

- Parameters:
  - VRP(WC) = (0.0, 0)
  - VPN(WC) = (0.0, 1)
  - VUP(WC) = (0.1, 0)
  - PRP(VRC) = (8.10, 100)
  - window(VRC) = (-1.17, -1.17)
  - projection = parallel
Programming assignment 4

- Read SMF file
- Implement parallel projection
- Implement perspective projection
- Output projected and clipped polygon edges