Overview

- 3D solid model representations
  - Implicit models
  - Superquadrics
  - Blobs/spheres
  - Swept objects
  - Boundary representations
  - Spatial enumerations
  - Distance fields
  - Quadtree/octree
  - Stochastic models

Implicit Solid Modeling

- Idea:
  - Represents solid as the set of points where an implicit global function takes on certain values
    - Usually:
      - \( F(x,y,z) < 0 \), points inside of object
      - \( F(x,y,z) = 0 \), points on object’s surface
      - \( F(x,y,z) > 0 \), points outside of object
  - Primitive solids are combined using CSG
  - Composition operations are implemented by functionals which provide an implicit function for the resulting solid

Quadratic Surfaces

- Sphere
  \( x^2 + y^2 + z^2 = r^2 \)

- Ellipsoid
  \[ \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \]

- Torus
  \[ \left( \sqrt{x^2 + y^2} - R \right)^2 + z^2 = r^2 \]

- General form
  \[ \sum_{i=1}^{n} a_i x^i + \sum_{i=1}^{n} b_i y^i + \sum_{i=1}^{n} c_i z^i + \sum_{i=1}^{n} d_i \cdot x^i \cdot y^i + \sum_{i=1}^{n} e_i \cdot y^i \cdot z^i + \sum_{i=1}^{n} f_i \cdot z^i \cdot x^i + \sum_{i=1}^{n} g_i \cdot x^i \cdot y^i \cdot z^i + d = 0 \]

Superellipsoid Surfaces

- Generalization of ellipsoid
- Control parameters \( s_1 \) and \( s_2 \):
  \[ \left( \frac{x^2}{s_1} \right)^{1/n} + \left( \frac{y^2}{s_2} \right)^{1/n} = 1 \]
- If \( s_1 = s_2 = 1 \) then regular ellipsoid
- Has an implicit and parametric form!

CSG with Superquadrics
CSG with Superellipsoids

Blobby Objects
- Do not maintain shape, topology
  - Water drops
  - Molecules
  - Force fields
- But can maintain other properties, like volume

Gaussian Bumps
- Model object as a sum of Gaussian bumps/blobs
  \[ f(x, y, z) = \sum b_k e^{-\frac{r_k^2}{2T^2}} = 0 \]
- Where \( r_k^2 = x_k^2 + y_k^2 + z_k^2 \) and \( T \) is a threshold.

Metaballs (Blinn Blobbies)

Ray-traced Metaballs

Implicit Modeling System
U. of Calgary
- Combine “primitives”
  - Points, lines, planes, polygons, cylinders, ellipsoids
- Calculate field around primitives
- View iso-surface of implicit function
Implicit Modeling System
U. of Calgary
Can apply blends and warps

Sweep Representations
• An alternative way to represent a 3D object
• Idea
  – Given a primitive (e.g., polygon, sphere)
  – And a sweep (e.g., vector, curve…)
  – Define solid as space swept out by primitive

Sweep Representations
• Issues:
  – How to generate resulting surface?
  – What about self-intersections?
  – How to define intersection?

Approximate Representations
• Idea: discretize the world!
• Surface Models
  – Mesh, facet, and polygon representations
• Volume Models
  – Spatial enumeration
  – Voxelization

Examples
• From exact to facets….

Boundary Representation
Solid Modeling
• The de facto standard for CAD since ~1987
  – BReps integrated into CAGD surfaces + analytic surfaces +
    boolean modeling
• Models are defined by their boundaries
• Topological and geometric integrity constraints are
  enforced for the boundaries
  – Faces meet at shared edges, vertices are shared, etc.
Let's Start Simple: Polyhedral Solid Modeling

- Definition
  - Solid bounded by polygons whose edges are each a member of an even number of polygons
  - A 2-manifold: edges members of 2 polygons

Properties of 2-Manifolds

- For any point on the boundary, its neighborhood is a topological 2D disk
- If not a 2-manifold, neighborhood not a disk

Euler's Formula

- For simple polyhedron (no holes):
  \[ V - E + F = 2 \]
- If formula is true the surface is closed

Euler's Formula (Generalized)

- Genus is the number of holes through the object
- Euler Operators have been the basis of several modeling systems (Mantyla et al.)

Euler Operators

- Representation
  - Points, Lines/Edges, Polygons
- Modeling
  - Generalization of 3D clipping to non-convex polyhedra, enables implementation of booleans

Steps to Creating a Polyhedral Solid Modeler
State of the Art: BRep Solid Modeling
• … but much more than polyhedra
• Two main (commercial) alternatives
  – All NURBS, all the time
    • Pro/E, SDRC, …
  – Analytic surfaces + parametric surfaces + NURBS + …. all stitched together at edges
    • Parasolid, ACIS, …

Issues in Boundary Representation Solid Modeling
• Very complex data structures
  – NURBS-based winged-edges, etc
• Complex algorithms
  – manipulation, boolean, collision detection
• Robustness
• Integrity
• Translation
• Features
• Constraints and Parametrics

Other Issues: Non-Manifold Solids
• There are cases where you may need to model entities that are not entirely 3D

Cell Decomposition
• Set of primitive cells
• Parameterized
• Often curved
• Compose complex objects by gluing cells together
• Used in finite-element analysis

Spatial Occupancy Enumeration
• Brute force
  – A grid
• Pixels
  – Picture elements
• Voxel
  – Volume elements
• Quadtree
  – 2D adaptive representation
• Octree
  – 3D adaptive representation
  – Extension of quadtree

Brute Force Spatial Occupancy Enumeration
• Impose a 2D/3D grid
  – Like graph paper or sugar cubes
• Identify occupied cells
• Problems
  – High fidelity requires many cells
• “Modified”
  – Partial occupancy
Distance Volume

- Store signed distance to surface at each voxel

Narrow-band representation

Iso-surface at value 0 approximates the original surface.

Offset Surfaces from Distance Volumes

Quadtree

- Hierarchically represent spatial occupancy
- Tree with four regions
  - NE, NW, SE, SW
  - “dark” if occupied

Quadtree Data Structure

Foley/VanDam, 1990/1994

Octree

- 8 octants 3D space
  - Left, Right, Up,
    Down, Front, Back

Boolean Operations on Octrees

Foley/VanDam, 1990/1994
Adaptive Distance Fields

- Quadtrees/Octrees that store distances

Applications for Spatial Occupancy Enumeration

- Many different applications
  - GIS
  - Medical
  - Engineering Simulation
  - Volume Rendering
  - Video Gaming
  - Approximating real-world data

Issues with Spatial Occupancy Enumeration

- Approximate
  - Kind of like faceting a surface, discretizing 3D space
  - Operationally, the combinatorics (as opposed to the numerics) can be challenging
  - Not as good for applications wanting exact computation (e.g. tool path programming)

Binary Space Partition Trees (BSP Trees)

- Recursively divide space into subspaces
- Arbitrary orientation and position of planes
- Homogeneous regions are leaves called in/out cells

Statistical Representations

- Store density (material vs. void)
- Statistical description of geometry
- Goal – describe the porosity without storing the geometry information

Generated by Termite Agents Simulation.

Stochastic Geometry

- Need some way of converting a solid into some representative statistical form
- From each material voxel, calculate the distance to the nearest voxel that is not material
- Repeat for void voxels
- Store distributions:
  - one for empty space
  - one for material
  - density value
Application: Biological Models
- Bone tissue
- MRI data
- Other biological data
- Solid modeling

Application: Surface Texture

Programming Assignment 5
- Extend XPM to 60 different RGB colors
- Read 3 models and assign each a color
- Implement Z-buffer rendering
- Implement front & back cutting planes
  - Only render parts of models between planes
- Implement linear depth-cueing
  - Color = base_color*(z-far)/(near-far)
- Re-use and extend 2D polygon filling

End