Overview

• 3D solid model representations
  – Implicit models
  – Super/quadrics
  – Blobbies
  – Swept objects
  – Boundary representations
  – Spatial enumerations
  – Distance fields
  – Quadtrees/octrees
  – Stochastic models
Implicit Solid Modeling

- Computer Algebra meets CAD
- Idea:
  - Represents solid as the set of points where an implicit global function takes on certain value
    - $F(x,y,z) < \text{val}$
  - Primitive solids are combined using CSG
  - Composition operations are implemented by functionals which provide an implicit function for the resulting solid

From M. Ganter, D. Storti, G. Turkiyyah @ UW
Quadratic Surfaces

- **Sphere**
  
  \[ x^2 + y^2 + z^2 = r^2 \]

- **Ellipsoid**
  
  \[
  \frac{x^2}{r_x^2} + \frac{y^2}{r_y^2} + \frac{z^2}{r_z^2} = 1
  \]

- **Torus**
  
  \[
  \sqrt{\left( \frac{x}{r_x} \right)^2 + \left( \frac{y}{r_y} \right)^2 + \left( \frac{z}{r_z} \right)^2} = 1
  \]

- **General form**
  
  \[
  a \cdot x^2 + b \cdot y^2 + c \cdot z^2 + 2f \cdot yz + 2g \cdot xz + 2h \cdot xy + 2p \cdot x + 2q \cdot y + 2r \cdot z + d = 0
  \]
Superellipsoid Surfaces

- Generalization of ellipsoid
- Control parameters $s_1$ and $s_2$

\[
x \left(\frac{x}{r_x}\right)^{s_2} + y \left(\frac{y}{r_y}\right)^{s_2} + z \left(\frac{z}{r_z}\right)^{s_1} = 1
\]

- If $s_1 = s_2 = 1$ then regular ellipsoid
- Has an implicit and parametric form!
CSG with Superquadrics
CSG with Superellipsoids
Bobbyy Objects

- Do not maintain shape, topology
  - Water drops
  - Molecules
  - Force fields
- But can maintain other properties, like volume
Gaussian Bumps

- Model object as a sum of Gaussian bumps/blobs

\[ f(x, y, z) = \sum_{k} b_k e^{-a_k r_k^2} \quad \square T = 0 \]

- Where \( r_k^2 = x_k^2 + y_k^2 + z_k^2 \) and \( T \) is a threshold.
Metaballs (Blinn Blobbies)
Ray-traced Metaballs
Implicit Modeling System
U. of Calgary

- Combine “primitives”
  - Points, lines, planes, polygons, cylinders, ellipsoids
- Calculate field around primitives
- View Iso-surface of implicit function
Implicit Modeling System
U. of Calgary

Can apply blends and warps
Sweep Representations

- An alternative way to represent a 3D object
- Idea
  - Given a primitive (e.g. polygon, sphere)
  - And a sweep (e.g. vector, curve...)
  - Define solid as space swept out by primitive

Foley/VanDam, 1990/1994
Sweep Representations

• Issues:
  – How to generate resulting surface?
  – What about self-intersections?
  – How to define intersection?
Approximate Representations

• Idea: discretize the world!
• Surface Models
  – Mesh, facet and polygon representations
• Volume Models
  – spatial enumeration
  – voxelization
Examples

• From exact to facets....
Boundary Representation
Solid Modeling

- The de facto standard for CAD since ~1987
  - BReps integrated into CAGD surfaces + analytic surfaces + boolean modeling
- Models are defined by their boundaries
- Topological and geometric integrity constraints are enforced for the boundaries
  - Faces meet at shared edges, vertices are shared, etc.
Let’s Start Simple: Polyhedral Solid Modeling

• Definition
  – Solid bounded by polygons whose edges are each a member of an even number of polygons
  – A 2-manifold: edges members of 2 polygons
Properties of 2-Manifolds

- For any point on the boundary, its neighborhood is a topological 2D disk
- If not a 2-manifold, neighborhood not a disk
Euler’s Formula

- For simple polyhedra (no holes):
  \[ \text{#Vertices} - \text{#Edges} + \text{#Faces} = 2 \]

![Cube](V=8, E=12, F=6)

![Pyramid](V=5, E=8, F=5)

![Truncated Tetrahedron](V=6, E=12, F=8)
Euler’s Formula (Generalized)

#Vertices - #Edges + #Faces - #Holes_in_faces = 2 (#Components – Genus)

- Genus is the # holes through the object
- Euler Operators have been the basis of several modeling systems (Mantyla et al.)

\[ V - E + F - H = 2(C - G) \]

| 24 | 36 | 15 | 3 | 1 | 1 |
## Euler Operators

<table>
<thead>
<tr>
<th>Operator Name</th>
<th>Meaning</th>
<th>V</th>
<th>E</th>
<th>F</th>
<th>L</th>
<th>S</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>MEV</td>
<td>Make an edge and a vertex</td>
<td>+1</td>
<td>+1</td>
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<td></td>
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<tr>
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<tr>
<td>MSFV</td>
<td>Make a shell, a face and a vertex</td>
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<td></td>
<td>+1</td>
<td>+1</td>
<td>+1</td>
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<tr>
<td>MSG</td>
<td>Make a shell and a hole</td>
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<td></td>
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<tr>
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<td></td>
<td></td>
<td>-1</td>
<td></td>
</tr>
</tbody>
</table>

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</tr>
</thead>
<tbody>
<tr>
<td>KEV</td>
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<tr>
<td>KSFV</td>
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<td>-1</td>
<td>-1</td>
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<tr>
<td>KEML</td>
<td>Kill an edge and make a loop</td>
<td>-1</td>
<td></td>
<td></td>
<td>+1</td>
<td></td>
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</tr>
</tbody>
</table>

Loop $L \rightarrow H$, Shell $S \rightarrow C$
Steps to Creating a Polyhedral Solid Modeler

• Representation
  – Points, Lines/Edges, Polygons

• Modeling
  – Generalization of 3D clipping to non-convex polyhedra, enables implementation of booleans
State of the Art: BRep Solid Modeling

• … but much more than polyhedra
• Two main (commercial) alternatives
  – All NURBS, all the time
    • Pro/E, SDRC, …
  – Analytic surfaces + parametric surfaces + NURBS + …. all stitched together at edges
    • Parasolid, ACIS, …
Issues in Boundary Representation Solid Modeling

• Very complex data structures
  – NURBS-based winged-edges, etc
• Complex algorithms
  – manipulation, booleans, collision detection
• Robustness
• Integrity
• Translation
• Features
• Constraints and Parametrics
Other Issues: Non-Manifold Solids

- There are cases where you may need to model entities that are not entirely 3D
Cell Decomposition

- Set of primitive cells
- Parameterized
- Often curved
- Compose complex objects by gluing cells together
- Used in finite-element analysis
Spatial Occupancy Enumeration

- Brute force
  - A grid

- Pixels
  - Picture elements

- Voxels
  - Volume elements

- Quadtrees
  - 2D adaptive representation

- Octrees
  - 3D adaptive representation
  - Extension of quadtrees
Brute Force Spatial Occupancy Enumeration

- Impose a 2D/3D grid
  - Like graph paper or sugar cubes
- Identify occupied cells
- Problems
  - High fidelity requires many cells
- “Modified”
  - Partial occupancy

Foley/VanDam, 1990/1994
Distance Volume

• Store signed distance to surface at each voxel

Iso-surface at value 0 approximates the original surface.
Offset Surfaces from Distance Volumes
Quadtree

- Hierarchically represent spatial occupancy
- Tree with four regions
  - NE, NW, SE, SW
  - “dark” if occupied
Quadtree Data Structure

F = full
P = partially full
E = empty

Quadrant numbering

Foley/VanDam, 1990/1994
Octree

- 8 octants 3D space
  - Left, Right, Up, Down, Front, Back
Boolean Operations on Octrees

S \bigcap T

S \bigcap T

Foley/VanDam, 1990/1994
Adaptive Distance Fields

- Quadtrees/Octrees that store distances
Applications for Spatial Occupancy Enumeration

- Many different applications
  - GIS
  - Medical
  - Engineering Simulation
  - Volume Rendering
  - Video Gaming
  - Approximating real-world data
  - ....
Issues with Spatial Occupancy Enumeration

• Approximate
  – Kind of like faceting a surface, discretizing 3D space
  – Operationally, the combinatorics (as opposed to the numerics) can be challenging
  – Not as good for applications wanting exact computation (e.g. tool path programming)
Binary Space Partition Trees (BSP Trees)

- Recursively divide space into subspaces
- Arbitrary orientation and position of planes
- Homogeneous regions are leaves called in/out cells

Foley/VanDam, 1990/1994
Statistical Representations

• Store density (material vs. void)
• Statistical description of geometry
• Goal – describe the porosity without storing the geometry information
Stochastic Geometry

• Need some way of converting a solid into some representative statistical form
• From each material voxel, calculate the distance to the nearest voxel that is not material
• Repeat for void voxels
• Store distributions:
  – one for empty space
  – one for material
  – density value
Application: Biological Models

- Bone tissue
- MRI data
- Other biological data
- Solid modeling

MRI scan of left shoulder

Bone matrix from scanned data
Application: Surface Texture
Application: Surface Texture
Programming Assignment 5

- Extend XPM to 60 different RGB colors
- Read 3 models and assign each a color
- Implement Z-buffer rendering
- Implement front & back cutting planes
  - Only render parts of models between planes
- Implement linear depth-cueing
  - Color = \( \text{base\_color} \times \frac{z\text{-far}}{\text{near\text{-}far}} \)
- Re-use and extend 2D polygon filling
End