Hierarchical Modeling

CS 432 Interactive Computer Graphics
Prof. David E. Breen
Department of Computer Science
Objectives

• Examine the limitations of linear modeling
  - Symbols and instances
• Introduce hierarchical models
  - Articulated models
  - Robots
• Introduce Tree and DAG models
Instance Transformation

• Start with a prototype object (a symbol)
• Each appearance of the object in the model is an instance
  - Must scale, orient, position
  - Defines instance transformation
Symbol-Instance Table

Can store a model by assigning a number to each symbol and storing the parameters for the instance transformation.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Scale</th>
<th>Rotate</th>
<th>Translate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$s_x$, $s_y$, $s_z$</td>
<td>$\theta_x$, $\theta_y$, $\theta_z$</td>
<td>$d_x$, $d_y$, $d_z$</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Relationships in Car Model

• Symbol-instance table does not show relationships between parts of model

• Consider model of car
  - Chassis + 4 identical wheels
  - Two symbols

• Rate of forward motion determined by rotational speed of wheels
Structure Through Function Calls

car(speed)
{
    chassis()
    wheel(right_front);
    wheel(left_front);
    wheel(right_rear);
    wheel(left_rear);
}

• Fails to show relationships well
• Look at problem using a graph
Graphs

- Set of nodes and edges (links)
- Edge connects a pair of nodes
  - Directed or undirected
- Cycle: directed path that is a loop
Tree

- Graph in which each node (except the root) has exactly one parent node
  - May have multiple children
  - Leaf or terminal node: no children
Tree Model of Car

- Chassis
  - Right-front wheel
  - Left-front wheel
  - Right-rear wheel
  - Left-rear wheel
DAG Model

- If we use the fact that all the wheels are identical, we get a *directed acyclic graph* - Not much different than dealing with a tree
Modeling with Trees

• Must decide what information to place in nodes and what to put in edges

• Nodes
  - What to draw
  - Pointers to children

• Edges
  - May have information on incremental changes to transformation matrices (can also store in nodes)
Transformations to Change Coordinate Systems

• Issue: the world has many different relative frames of reference
• How do we transform among them?
• Example: CAD Assemblies & Animation Models
Transformations to Change Coordinate Systems

- 4 coordinate systems
- 1 point $P$

$M_{1\leftarrow 2} = T(4,2)$
$M_{2\leftarrow 3} = T(2,3) \cdot S(0.5,0.5)$
$M_{3\leftarrow 4} = T(6.7,1.8) \cdot R(45^\circ)$

$M_{i\leftarrow k} = M_{i\leftarrow j} \cdot M_{j\leftarrow k}$
Coordinate System Example

(1)

• Translate the House to the origin

\[ M_{1\leftarrow 2} = T(x_1, y_1) \]
\[ M_{2\leftarrow 1} = (M_{1\leftarrow 2})^{-1} \]
\[ = T(-x_1, -y_1) \]

The matrix \( M_{ij} \) that maps points from coordinate system \( j \) to \( i \) is the inverse of the matrix \( M_{ji} \) that maps points from coordinate system \( j \) to coordinate system \( i \).
• Transformation Composition:

\[ M_{5\leftarrow 1} = M_{5\leftarrow 4} \cdot M_{4\leftarrow 3} \cdot M_{3\leftarrow 2} \cdot M_{2\leftarrow 1} \]
World Coordinates and Local Coordinates

• To move the tricycle, we need to know how all of its parts relate to the WCS

• Example: front wheel rotates on the ground wrt the front wheel’s z axis:

  \[ P^{(wo)} = T(\alpha r, 0, 0) \cdot R_z(\alpha) \cdot P^{(wh)} \]

  Coordinates of \( P \) in wheel coordinate system:

  \[ P^{(wh)} = R_z(\alpha) \cdot P^{(wh)} \]
Robot Arm

robot arm

parts in their own coordinate systems
Articulated Models

• Robot arm is an example of an *articulated model*
  - Parts connected at joints
  - Can specify state of model by giving all joint angles
Relationships in Robot Arm

• Base rotates independently
  - Single angle determines position

• Lower arm attached to base
  - Its position depends on rotation of base
  - Must also translate relative to base and rotate about connecting joint

• Upper arm attached to lower arm
  - Its position depends on both base and lower arm
  - Must translate relative to lower arm and rotate about joint connecting to lower arm
Required Matrices

• Rotation of base: \( \mathbf{R}_b \)
  - Apply \( \mathbf{M} = \mathbf{R}_b \) to base

• Translate lower arm relative to base: \( \mathbf{T}_{lu} \)

• Rotate lower arm around joint: \( \mathbf{R}_{lu} \)
  - Apply \( \mathbf{M} = \mathbf{R}_b \mathbf{T}_{lu} \mathbf{R}_{lu} \) to lower arm

• Translate upper arm relative to lower arm: \( \mathbf{T}_{uu} \)

• Rotate upper arm around joint: \( \mathbf{R}_{uu} \)
  - Apply \( \mathbf{M} = \mathbf{R}_b \mathbf{T}_{lu} \mathbf{R}_{lu} \mathbf{T}_{uu} \mathbf{R}_{uu} \) to upper arm
var render = function() {
    gl.clear(gl.COLOR_BUFFER_BIT | gl.DEPTH_BUFFER_BIT);
    modelViewMatrix = rotate(theta[Base], 0, 1, 0);
    base();
    modelViewMatrix = mult(modelViewMatrix, translate(0.0, BASE_HEIGHT, 0.0));
    modelViewMatrix = mult(modelViewMatrix, rotate(theta[LowerArm], 0, 0, 1));
    lowerArm();
    modelViewMatrix = mult(modelViewMatrix, translate(0.0, LOWER_ARM_HEIGHT, 0.0));
    modelViewMatrix = mult(modelViewMatrix, rotate(theta[UpperArm], 0, 0, 1));
    upperArm();
    requestAnimFrame(render);
}
OpenGL Code for Robot

- At each level of hierarchy, calculate \texttt{ModelView} matrix in application.
- Send matrix to shaders
- Draw geometry for one level of hierarchy
- Apply \texttt{ModelView} matrix in shader
Tree Model of Robot

- Note code shows relationships between parts of model
  - Can change “look” of parts easily without altering relationships
- Simple example of tree model
- Want a general node structure for nodes
Possible Node Structure

- Code for drawing part or pointer to drawing function
- Linked list of pointers to children
- Matrix relating node to parent
Generalizations

• Need to deal with multiple children
  - How do we represent a more general tree?
  - How do we traverse such a data structure?

• Animation
  - How to use dynamically?
  - Can we create and delete nodes during execution?
Objectives

• Build a tree-structured model of a humanoid figure
• Examine various traversal strategies
• Build a generalized tree-model structure that is independent of the particular model
Humanoid Figure
Building the Model

- Can build a simple implementation using quadrics: ellipsoids and cylinders
- Access parts through functions
  - `torso()`
  - `left_upper_arm()`
- Matrices describe position of node with respect to its parent
  - $\mathbf{M}_{lla}$ positions left lower arm with respect to left upper arm
Tree with Matrices
Display and Traversal

- The position of the figure is determined by 11 joint angles (two for the head and one for each other part)
- Display of the tree requires a graph traversal
  - Visit each node once
  - Display function at each node that describes the part associated with the node, applying the correct transformation matrix for position and orientation
Transformation Matrices

• There are 10 relevant matrices
  - $M$ positions and orients entire figure through the torso which is the root node
  - $M_h$ positions head with respect to torso
  - $M_{lua}, M_{rua}, M_{lul}, M_{rul}$ position arms and legs with respect to torso
  - $M_{lila}, M_{rla}, M_{lll}, M_{rll}$ position lower parts of limbs with respect to corresponding upper limbs
Stack-based Traversal

• Set model-view matrix to $M$ and draw torso
• Set model-view matrix to $MM_h$ and draw head
• For left-upper arm need $MM_{lua}$ and so on
• Rather than recomputing $MM_{lua}$ from scratch or using an inverse matrix, we can use the matrix stack to store $M$ and other matrices as we traverse the tree
Traversal Code

```cpp
figure() {
    PushMatrix();
    torso();
    Rotate (...);
    head();
    PopMatrix();
    PushMatrix();
    Translate (...);
    Rotate (...);
    left_upper_arm();
    PopMatrix();
    PushMatrix();
}
```

- save present current xform matrix
- update ctm for head
- recover original ctm
- save it again
- update ctm for left upper arm
- recover and save original ctm again
- rest of code
Analysis

• The code describes a particular tree and a particular traversal strategy
  - Can we develop a more general approach?
• Note that the sample code does not include state changes, such as changes to colors
  - May also want to push and pop other attributes to protect against unexpected state changes affecting later parts of the code
General Tree Data Structure

• Need a data structure to represent tree and an algorithm to traverse the tree

• We will use a left-child right sibling structure
  - Uses linked lists
  - Each node in data structure is two pointers
  - Left: linked list of children
  - Right: next node (i.e. siblings)
Left-Child Right-Sibling Tree
Tree node Structure

• At each node we need to store
  - Pointer to sibling
  - Pointer to child
  - Pointer to a function that draws the object represented by the node
  - Homogeneous coordinate matrix to multiply on the right of the current model-view matrix
    • Represents changes going from parent to node
    • In WebGL this matrix is a 1D array storing matrix by columns
function createNode(transform, render, sibling, child) {
    var node = {
        transform: transform,
        render: render,
        sibling: sibling,
        child: child,
    }
    return node;
}
function initNodes(Id) {
    var m = mat4();
    switch(Id) {
    case torsoId:
        m = rotate(theta[torsoId], 0, 1, 0);
        figure[torsoId] = createNode( m, torso, null, headId );
        break;
    case head1Id:
    case head2Id:
        m = translate(0.0, torsoHeight+0.5*headHeight, 0.0);
        m = mult(m, rotate(theta[head1Id], 1, 0, 0));
        m = mult(m, rotate(theta[head2Id], 0, 1, 0));
        m = mult(m, translate(0.0, -0.5*headHeight, 0.0));
        figure[headId] = createNode( m, head, leftUpperArmId, null);
        break;
    }
• The position of figure is determined by 11 joint angles stored in theta[11]
• Animate by changing the angles and redisplaying
• We form the required matrices using rotate and translate
• Because the matrix is formed using the model-view matrix, we may want to first push original model-view matrix on matrix stack
function traverse(Id) {
    if(Id == null) return;
    stack.push(modelViewMatrix);
    modelViewMatrix = mult(modelViewMatrix, figure[Id].transform);
    figure[Id].render();
    if(figure[Id].child != null) traverse(figure[Id].child);
    modelViewMatrix = stack.pop();
    if(figure[Id].sibling != null) traverse(figure[Id].sibling);
}

var render = function() {
    gl.clear( gl.COLOR_BUFFER_BIT );
    traverse(torsoldId);
    requestAnimFrame(render);
}
Traversing Code & Matrices

- `figure()` called with CTM set
- $M_{fig}$ defines figure's place in world

```cpp
figure() {
    PushMatrix();
    torso();
    Rotate (...);
    head();
    PopMatrix();
    PushMatrix();
    Translate(...);
    Rotate(...);
    left_upper_arm();
}
```

<table>
<thead>
<tr>
<th>Stack</th>
<th>CTM</th>
<th>$M_{fig}$</th>
<th>$M_{figM_h}$</th>
<th>$M_{figM_{lua}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Code</td>
<td>Stack</td>
<td>CTM</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-----------------------------</td>
<td>---------------------</td>
<td>----------------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td><code>PushMatrix()</code></td>
<td>Stack</td>
<td>CTM</td>
<td></td>
<td></td>
</tr>
<tr>
<td><code>Translate(...);</code></td>
<td><code>M_{fig}M_{lua}</code></td>
<td><code>M_{fig}M_{lua}</code></td>
<td></td>
<td></td>
</tr>
<tr>
<td><code>Rotate(...);</code></td>
<td><code>M_{fig}</code></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><code>left_lower_arm();</code></td>
<td><code>Stack</code></td>
<td><code>CTM</code></td>
<td></td>
<td></td>
</tr>
<tr>
<td><code>PopMatrix();</code></td>
<td><code>M_{fig}M_{lua}</code></td>
<td><code>M_{fig}M_{lua}M_{lla}</code></td>
<td></td>
<td></td>
</tr>
<tr>
<td><code>PopMatrix();</code></td>
<td><code>M_{fig}</code></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><code>PushMatrix()</code></td>
<td><code>Stack</code></td>
<td><code>CTM</code></td>
<td></td>
<td></td>
</tr>
<tr>
<td><code>Translate(...);</code></td>
<td><code>M_{fig}</code></td>
<td><code>M_{fig}M_{lua}</code></td>
<td></td>
<td></td>
</tr>
<tr>
<td><code>Rotate(...);</code></td>
<td><code>Stack</code></td>
<td><code>CTM</code></td>
<td></td>
<td></td>
</tr>
<tr>
<td><code>right_upper_arm();</code></td>
<td><code>Stack</code></td>
<td><code>CTM</code></td>
<td></td>
<td></td>
</tr>
<tr>
<td>...</td>
<td><code>M_{fig}</code></td>
<td><code>M_{fig}</code></td>
<td></td>
<td></td>
</tr>
<tr>
<td>...</td>
<td><code>Stack</code></td>
<td><code>CTM</code></td>
<td></td>
<td></td>
</tr>
<tr>
<td>...</td>
<td><code>M_{fig}</code></td>
<td><code>M_{fig}M_{rua}</code></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Notes

• We must save model-view matrix before multiplying it by node matrix
  - Updated matrix applies to children of node but not to siblings which contain their own matrices

• The traversal program applies to any left-child right-sibling tree
  - The particular tree is encoded in the definition of the individual nodes

• The order of traversal matters because of possible state changes in the functions
Dynamic Trees

- Because we are using JS, the nodes and the node structure can be changed during execution.
- Definition of nodes and traversal are essentially the same as before but we can add and delete nodes during execution.
- In desktop OpenGL, if we use pointers, the structure can be dynamic.
Solid modeling introduces a mathematical theory of solid shape
- Domain of objects
- Set of operations on the domain of objects
- Representation that is
  - Unambiguous
  - Accurate
  - Unique
  - Compact
  - Efficient
Solid Objects and Operations

- Solids are point sets
  - Boundary and interior
- Point sets can be operated on with boolean algebra (union, intersect, etc)
Constructive Solid Geometry (CSG)

- A tree structure combining primitives via regularized boolean operations
- Primitives can be solids or half spaces
A Sequence of Boolean Operations

- Boolean operations
- Rigid transformations
The Induced CSG Tree
The Induced CSG Tree

- Can also be represented as a directed acyclic graph (DAG)
Issues with Constructive Solid Geometry

• Non-uniqueness
• Choice of primitives
• How to handle more complex modeling?
  - Sculpted surfaces? Deformable objects?
Issues with Constructive Solid Geometry

• Non-Uniqueness
  - There is more than one way to model the same artifact
  - Hard to tell if A and B are identical
Issues with CSG

- Minor changes in primitive objects greatly affect outcomes
- Shift up top solid face
Uses of Constructive Solid Geometry

- Found (basically) in every CAD system
- Elegant, conceptually and algorithmically appealing
- Good for
  - Rendering, ray tracing, simulation
  - BRL CAD
Graphical Objects and Scene Graphs
Objectives

• Introduce graphical objects
• Generalize the notion of objects to include lights, cameras, attributes
• Introduce scene graphs
• three.js (threejs.org)
Limitations of Immediate Mode Graphics

- When we define a geometric object in an application, upon execution of the code the object is passed through the pipeline.
- It then disappeared from the graphical system.
- To redraw the object, either changed or the same, we had to reexecute the code.
- Display lists provided only a partial solution to this problem.
Retained Mode Graphics

• Display lists were server side
• GPUs allowed data to be stored on GPU
• Essentially all immediate mode functions have been deprecated
• Nevertheless, OpenGL is a low level API
OpenGL and Objects

• OpenGL lacks an object orientation
• Consider, for example, a green sphere
  - We can model the sphere with polygons
  - Its color is determined by the OpenGL state and is not a property of the object
  - Loose linkage with vertex attributes
• Defies our notion of a physical object
• We can try to build better objects in code using object-oriented languages/techniques
Imperative Programming Model

• Example: rotate a cube

• The rotation function must know how the cube is represented
  - Vertex list
  - Edge list
Object-Oriented Programming Model

• In this model, the representation is stored with the object

  Application → Cube Object

  message

• The application sends a *message* to the object
• The object contains functions (*methods*) which allow it to transform itself
C/C++/Java/JS

- Can try to use C structs to build objects
- C++/Java/JS provide better support
  - Use class construct
  - With C++ we can hide implementation using public, private, and protected members
  - JS provides multiple methods for object
Cube Object

• Suppose that we want to create a simple cube object that we can scale, orient, position and set its color directly through code such as

```javascript
var mycube = new Cube();
mycube.color[0]=1.0;
mycube.color[1]= mycube.color[2]=0.0;
mycube.matrix[0][0]=........
```
Cube Object Functions

• We would also like to have functions that act on the cube such as
  - `mycube.translate(1.0, 0.0, 0.0);`
  - `mycube.rotate(theta, 1.0, 0.0, 0.0);`
  - `setcolor(mycube, 1.0, 0.0, 0.0);`

• We also need a way of displaying the cube
  - `mycube.render();`
Building the Cube Object

```javascript
var cube {
    var color[3];
    var matrix[4][4];
}
```
The Implementation

• Can use any implementation in the private part such as a vertex list
• The private part has access to public members and the implementation of class methods can use any implementation without making it visible
• Render method is tricky but it will invoke the standard OpenGL drawing functions
Other Objects

• Other objects have geometric aspects
  - Cameras
  - Light sources

• But we should be able to have nongeometric objects too
  - Materials
  - Colors
  - Transformations (matrices)
cube mycube;

material plastic;
mycube.setMaterial(plastic);

camera frontView;
frontView.position(x, y, z);
JS Objects

• Can create much like Java or C++ objects
  - constructors
  - prototypes
  - methods
  - private methods and variables

```javascript
var myCube = new Cube();
myCube.color = [1.0, 0.0, 0.0];
myCube.instance = …….
```
Light Object

```javascript
var myLight = new Light();

// match Phong model

myLight.type = 0; // directional
myLight.position = ......;
myLight.orientation = ......;
myLight.specular = ......;
myLight.diffuse = ......;
myLight.ambient = ......;
```

Angel and Shreiner: Interactive Computer Graphics 7E © Addison-Wesley 2015
Scene Descriptions

• If we recall figure model, we saw that
  - We could describe model either by tree or by equivalent code
  - We could write a generic traversal to display

• If we can represent all the elements of a scene (cameras, lights, materials, geometry) as JS objects, we should be able to show them in a tree
  - Render scene by traversing this tree
Traversal

```java
myScene = new Scene();
myLight = new Light();
myLight.Color = ......;
...
myScene.Add(myLight);
object1 = new Object();
object1.color = ...
myScene.add(object1);
...
...
myScene.render();
```
Scene Graph History

• OpenGL development based largely on people who wanted to exploit hardware
  - real time graphics
  - animation and simulation
  - stand-alone applications

• CAD community needed to be able to share databases
  - real time not an issue and photorealism not issues
  - need cross-platform capability
  - first attempt: PHIGS
Scene Graph Organization

- Scene Graph
  - Scene Graph API
    - WebGL
    - OpenGL
    - Direct X
  - Database
    - WWW
Inventor and Java3D

- Inventor and Java3D provide a scene graph API
- Scene graphs can also be described by a file (text or binary)
  - Implementation independent way of transporting scenes
  - Supported by scene graph APIs
- However, primitives supported should match capabilities of graphics systems
  - Hence most scene graph APIs are built on top of OpenGL, WebGL or DirectX (for PCs)
• Want to have a scene graph that can be used over the World Wide Web
• Need links to other sites to support distributed data bases
• Virtual Reality Markup Language
  - Based on Inventor data base
  - Implemented with OpenGL
Open Scene Graph

• Supports very complex geometries by adding occlusion culling in first pass
• Supports translucently through a second pass that sorts the geometry
• First two passes yield a geometry list that is rendered by the pipeline in a third pass
three.js

- Popular scene graph built on top of WebGL
  - also supports other renderers
- See threejs.org
  - easy to download
  - many examples
- Also Eric Haines’ Udacity course
- Major differences in approaches to computer graphics
```
var scene = new THREE.Scene();
var camera = new THREE.PerspectiveCamera(75, window.innerWidth/
window.innerHeight, 0.1, 1000);

var renderer = new THREE.WebGLRenderer();
renderer.setSize(window.innerWidth, window.innerHeight);
document.body.appendChild(renderer.domElement);

var geometry = new THREE.CubeGeometry(1,1,1);
var material = new THREE.MeshBasicMaterial({color: 0x00ff00});
var cube = new THREE.Mesh(geometry, material);
scene.add(cube);
camera.position.z = 5;
```
var render = function () {
requestAnimationFrame(render);
cube.rotation.x += 0.1;
cube.rotation.y += 0.1;
renderer.render(scene, camera);
};
render();