CS 536
Computer Graphics

Solid Modeling

Week 7, Lecture 13

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Overview

• Solid Modeling
  – Boolean operations
  – Constructive Solid Geometry
Solids and Solid Modeling

• Solid modeling introduces a mathematical theory of solid shape
  – Domain of objects
  – Objects have a clearly-defined inside and outside
  – Set of operations on the domain of objects
  – Representation that is
    • Unambiguous
    • Accurate
    • Unique
    • Compact
    • Efficient
Solid Objects and Operations

- Solids are point sets
  - Boundary and interior
- Point sets can be operated on with boolean algebra (union, intersect, etc)
Solid Object Definitions

• Boundary points
  – Points where distance to the object and the object’s complement is zero

• Interior points
  – All the other points in the object

• Closure
  – Union of interior points and boundary points
Issues with 3D Set Operations

• Ops on 3D objects can create “non-3D objects” or objects with non-uniform dimensions
• Objects need to be “Regularized”
  – Take the closure of the interior

<table>
<thead>
<tr>
<th>Input set</th>
<th>Closure</th>
<th>Interior</th>
<th>Regularized</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
<td>(d)</td>
</tr>
</tbody>
</table>
Regularized Boolean Operations

• 3D Example
  – Two solids A and B
  – Intersection leaves a “dangling wall”
    • A 2D portion hanging off a 3D object
  – Closure of interior gives a uniform 3D result

Pics/Math courtesy of Dave Mount @ UMD-CP
Boolean Operations

- Other Examples:
- (c) ordinary intersection
- (d) regularized intersection
  - AB - objects on the same side
  - CD - objects on different sides
Boolean Operations
Constructive Solid Geometry (CSG)

- A tree structure combining primitives via regularized boolean operations
- Primitives can be solids or half spaces
A Sequence of Boolean Operations

- Boolean operations
- Rigid transformations
The Induced CSG Tree
The Induced CSG Tree

• Can also be represented as a directed acyclic graph (DAG)
Issues with Constructive Solid Geometry

• Non-uniqueness
• Choice of primitives
• How to handle more complex modeling?
  – Sculpted surfaces? Deformable objects?
Issues with Constructive Solid Geometry

• Non-Uniqueness
  – There is more than one way to model the same artifact
  – Hard to tell if A and B are identical
Issues with CSG

- Minor changes in primitive objects greatly affect outcomes
- Shift up top solid face

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Foley/VanDam, 1990/1994
Evaluating CSG Surface Points

• Given two solid primitives A and B, compute points on the surface of each primitive
• Test if points will be on the surface of evaluated CSG model A * B
• Use rules based on inside-outside status of the points relative to A and B
• Display points that follow rules
### CSG Surface Point Rules

<table>
<thead>
<tr>
<th>Operation</th>
<th>Conditions</th>
</tr>
</thead>
</table>
| \( A \cup B \) | On A and (On B || Outside of B)  
On B and (On A || Outside of A) |
| \( A \cap B \) | On A and (On B || Inside of B)   
On B and (On A || Inside of A) |
| \( A - B \)  | On A and (On B || Outside of B)    
On B and (On A || Inside of A) |
| \( B - A \)  | On A and (On B || Inside of B)     
On B and (On A || Outside of A) |

These are the points on the evaluated surface
### CSG Examples

<table>
<thead>
<tr>
<th>MANIFOLD INPUTS</th>
<th>UNION</th>
<th>INTERSECTION</th>
<th>DIFFERENCE</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A</strong></td>
<td><img src="image" alt="Union A B" /></td>
<td><img src="image" alt="Intersection A B" /></td>
<td><img src="image" alt="Difference A B" /></td>
</tr>
<tr>
<td><strong>B</strong></td>
<td><img src="image" alt="Union A B" /></td>
<td><img src="image" alt="Intersection A B" /></td>
<td><img src="image" alt="Difference A B" /></td>
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<tbody>
<tr>
<td><img src="image" alt="Regular Operation" /></td>
<td>1 body</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Uses of CSG
Constructive Solid Geometry

• Found (basically) in every CAD system
• Elegant, conceptually and algorithmically appealing
• Good for
  – Rendering, ray tracing, simulation
  – BRL CAD
CAD: Feature-Based Design

• CSG is the basic machinery behind CAD features

• Features are
  – Local modifications to object geom/topo with engineering significance
  – Often are additive or subtractive mods to shape
    • Hole, pocket, etc…
Parametric Modeling in CAD

- Feature relationships
- Constraints

(a) Gear
- Diam = 4.3
- Hub = 2.0
- Thickness = 0.5
- Teeth = 12
- Hole = 0.3

(b) Gear
- Diam = 6.0
- Hub = 1.0
- Thickness = 0.4
- Teeth = 18
- Hole = 0.3