Overview

- 3D solid model representations
  - Implicit models
  - Super/quadrics
  - Blobbies
  - Swept objects
  - Boundary representations
  - Spatial enumerations
  - Distance fields
  - Quadrees/octrees
  - Stochastic models

Implicit Solid Modeling

- Idea:
  - Represents solid as the set of points where an implicit global function takes on certain values
  - Usually
    - $F(x,y,z) < 0$, points inside of object
    - $F(x,y,z) = 0$, points on object’s surface
    - $F(x,y,z) > 0$, points outside of object
  - Primitive solids are combined using CSG
  - Composition operations are implemented by functionals which provide an implicit function for the resulting solid

Quadratic Surfaces

- Sphere
  $$x^2 + y^2 + z^2 = r^2$$
- Ellipsoid
  $$\frac{x^2}{s_1^2} + \frac{y^2}{s_2^2} + \frac{z^2}{s_3^2} = 1$$
- Torus
  $$\left(\frac{x}{r_1}\right)^2 + \left(\frac{y}{r_2}\right)^2 = 1$$
- General form
  $$a \cdot x^2 + b \cdot y^2 + c \cdot z^2 + 2f \cdot yz + 2g \cdot xz + 2h \cdot xy + 2p \cdot x + 2q \cdot y + 2r \cdot z + d = 0$$

Superellipsoid Surfaces

- Generalization of ellipsoid
- Control parameters $s_1$ and $s_2$
  $$\left[\left(\frac{x}{r_1}\right)^{\frac{1}{s_1}} + \left(\frac{y}{r_2}\right)^{\frac{1}{s_1}} + \left(\frac{z}{r_3}\right)^{\frac{1}{s_1}}\right]^{s_1} = 1$$
- If $s_1 = s_2 = 1$ then regular ellipsoid
- Has an implicit and a parametric form!
Superellipsoid Surfaces

- Normals defined by
  \[ n_x(u,v) = \frac{1}{A}c(v, 2 - s_1)c(u, 2 - s_2) \]
  \[ n_y(u,v) = \frac{1}{B}c(v, 2 - s_1)s(u, 2 - s_2) \]
  \[ n_z(u,v) = (1/C)s(v, 2 - s_1) \]

- A, B and C are scale factors of the X, Y & Z coordinates
- \( s_1 \) is the shape parameter for longitude lines
- \( s_2 \) is the shape parameter for latitude lines

Superellipsoid Inside-Outside Function

\[
F(x, y, z) = \left[ \left( \frac{x}{r_x} \right)^{2/s_2} + \left( \frac{y}{r_y} \right)^{2/s_2} \right]^{s_2/s_1} + \left( \frac{z}{r_z} \right)^{2/s_1} - 1
\]

CSG with Superquadrics

Blobby Objects

- Do not maintain shape, topology
  - Water drops
  - Molecules
  - Force fields
- But can maintain other properties, like volume
Gaussian Bumps

- Model object as a sum of Gaussian bumps/blobs
  \[ f(x, y, z) = \sum_{k} b_i e^{-\alpha \| r_k \|^2} - T = 0 \]
- Where \( r_k^2 = x_k^2 + y_k^2 + z_k^2 \) and \( T \) is a threshold.

Metaballs (Blinn Blobs)

Ray-traced Metaballs

Implicit Modeling System

- Combine "primitives"
  - Points, lines, planes, polygons, cylinders, ellipsoids
- Calculate field around primitives
- View iso-surface of implicit function

Implicit Modeling System

Sweep Representations

- An alternative way to represent a 3D object
- Idea
  - Given a primitive (e.g. polygon, sphere)
  - And a sweep (e.g. vector, curve…)
  - Define solid as space swept out by primitive
Sweep Representations

- Issues:
  - How to generate resulting surface?
  - What about self-intersections?
  - How to define intersection?

Foley/VanDam, 1990/1994

Approximate Representations

- Idea: discretize the world!
- Surface Models
  - Mesh, facet and polygon representations
- Volume Models
  - spatial enumeration
  - voxelization

Examples

- From exact to facets…

Pics/Math courtesy of Dave Mount @ UMD - CP

Boundary Representation Solid Modeling

- The de facto standard for CAD since ~1987
  - BReps integrated into CAGD surfaces + analytic surfaces + boolean modeling
- Models are defined by their boundaries
- Topological and geometric integrity constraints are enforced for the boundaries
  - Faces meet at shared edges, vertices are shared, etc.

Let’s Start Simple: Polyhedral Solid Modeling

- Definition
  - Solid bounded by polygons whose edges are each a member of an even number of polygons
  - A 2-manifold: edges members of 2 polygons

Properties of 2-Manifolds

- For any point on the boundary, its neighborhood is a topological 2D disk
- If not a 2-manifold, neighborhood not a disk
Euler’s Formula

- For simple polyhedron (no holes):
  \#Vertices - \#Edges + \#Faces = 2
- If formula is true the surface is closed

Euler’s Formula (Generalized)

- \#Vertices - \#Edges + \#Faces - \#Holes in faces = 2
- Genus is the \# holes through the object
- Euler Operators have been the basis of several modeling systems (Mantyla et al.)

Euler Operators

<table>
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<th>Operator Name</th>
<th>Meaning</th>
<th>V</th>
<th>E</th>
<th>F</th>
<th>L</th>
<th>S</th>
<th>G</th>
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<td>Make an edge and a vertex</td>
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Steps to Creating a Polyhedral Solid Modeler

- Representation
  - Points, Lines/Edges, Polygons
- Modeling
  - Generalization of 3D clipping to non-convex polyhedra, enables implementation of booleans

State of the Art: BRep Solid Modeling

- ... but much more than polyhedra
- Two main (commercial) alternatives
  - All NURBS, all the time
    - ProE, SDRC, ...
  - Analytic surfaces + parametric surfaces + NURBS + ... all stitched together at edges
    - Parasolid, ACIS, ...

Issues in Boundary Representation Solid Modeling

- Very complex data structures
  - NURBS-based winged-edges, etc
- Complex algorithms
  - manipulation, booleans, collision detection
- Robustness
- Integrity
- Translation
- Features
- Constraints and Parametrics
Other Issues: Non-Manifold Solids

- There are cases where you may need to model entities that are not entirely 3D

Spatial Occupancy Enumeration

- Brute force
  - A grid
- Pixels
  - Picture elements
- Voxels
  - Volume elements
- Quadtrees
  - 2D adaptive representation
- Octrees
  - 3D adaptive representation
  - Extension of quadtrees

Brute Force Spatial Occupancy Enumeration

- Impose a 2D/3D grid
  - Like graph paper or sugar cubes
- Identify occupied cells
- Problems
  - High fidelity requires many cells
- “Modified”
  - Partial occupancy

Distance Volume

- Store signed distance to surface at each voxel

Offset Surfaces from Distance Volumes

- Iso-surface at value 0 approximates the original surface.
**Quadtree**

- Hierarchically represent spatial occupancy
- Tree with four regions
  - NE, NW, SE, SW
  - “dark” if occupied

**Quadtree Data Structure**

- Hierarchical representation

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</tr>
<tr>
<td>2</td>
<td>3</td>
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</table>

Quad tree numbering

- F = full
- P = partially full
- E = empty

**Octree**

- 8 octants 3D space
  - Left, Right, Up, Down, Front, Back

**Boolean Operations on Octrees**

- $S \cup T$
- $S \cap T$

**Applications for Spatial Occupancy Enumeration**

- Many different applications
  - GIS
  - Medical
  - Engineering Simulation
  - Volume Rendering
  - Video Gaming
  - Approximating real-world data
  - ….
Issues with Spatial Occupancy Enumeration

- Approximate
  - Kind of like faceting a surface, discretizing 3D space
  - Operationally, the combinatorics (as opposed to the numerics) can be challenging
  - Not as good for applications wanting exact computation (e.g. tool path programming)

Binary Space Partition Trees (BSP Trees)

- Recursively divide space into subspaces
- Arbitrary orientation and position of planes
- Homogeneous regions are leaves called in/out cells

Statistical Representations

- Store density (material vs. void)
- Statistical description of geometry
- Goal – describe the porosity without storing the geometry information

Stochastic Geometry

- Need some way of converting a solid into some representative statistical form
- From each material voxel, calculate the distance to the nearest voxel that is not material
- Repeat for void voxels
- Store distributions:
  - one for empty space
  - one for material
  - density value

Application: Biological Models

- Bone tissue
- MRI data
- Other biological data
- Solid modeling

Distance vs. Probability

Generated by Termite Agents Simulation.

MRI scan of left shoulder

Bone matrix from scanned data
Application: Surface Texture

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Programming Assignment 4
- Implement parametric form of superellipsoids
- Iterate through u and v parameters
- Calculate point and normal for each (u,v) pair
- Only calculate one point at each of the poles
- Top and bottom rows should be a triangle fan with poles at center
- Other rows are quads that are broken into triangles
- Output mesh as Open Inventor

End