Hierarchical Models
Week 9, Lecture 19

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Objectives
• Examine the limitations of linear modeling
  - Symbols and instances
• Introduce hierarchical models
  - Articulated models
  - Robots
• Introduce Tree and DAG models

Instance Transformation
• Start with a prototype object (a symbol)
• Each appearance of the object in the model is an instance
  - Must scale, orient, position
  - Defines instance transformation

Symbol-Instance Table
Can store a model by assigning a number to each symbol and storing the parameters for the instance transformation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Scale</th>
<th>Rotate</th>
<th>Translate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

Relationships in Car Model
• Symbol-instance table does not show relationships between parts of model
• Consider model of car
  - Chassis + 4 identical wheels
  - Two symbols
  - Rate of forward motion determined by rotational speed of wheels

Structure Through Function Calls
```c
car(speed)
{
    chassis()
    wheel(right_front);
    wheel(left_front);
    wheel(right_rear);
    wheel(left_rear);
}
```
• Fails to show relationships well
• Look at problem using a graph
Graphs

- Set of nodes and edges (links)
- Edge connects a pair of nodes
  - Directed or undirected
- Cycle: directed path that is a loop

Tree

- Graph in which each node (except the root) has exactly one parent node
  - May have multiple children
  - Leaf or terminal node: no children

Tree Model of Car

- Chassis
  - Right-front wheel
  - Left-front wheel
  - Right-rear wheel
  - Left-rear wheel

DAG Model

- If we use the fact that all the wheels are identical, we get a directed acyclic graph
  - Not much different than dealing with a tree

Modeling with Trees

- Must decide what information to place in nodes and what to put in edges
- Nodes
  - What to draw
  - Pointers to children
- Edges
  - May have information on incremental changes to transformation matrices (can also store in nodes)

Robot Arm

- Arm and parts in their own coordinate systems
Articulated Models

- Robot arm is an example of an articulated model
  - Parts connected at joints
  - Can specify state of model by giving all joint angles

Relationships in Robot Arm

- Base rotates independently
  - Single angle determines position
- Lower arm attached to base
  - Its position depends on rotation of base
  - Must also translate relative to base and rotate about connecting joint
- Upper arm attached to lower arm
  - Its position depends on both base and lower arm
  - Must translate relative to lower arm and rotate about joint connecting to lower arm

Required Matrices

- Rotation of base: \( R_b \)
  - Apply \( M = R_b \) to base
- Translate lower arm relative to base: \( T_{lu} \)
- Rotate lower arm around joint: \( R_{lu} \)
  - Apply \( M = R_b T_{lu} R_{lu} \) to lower arm
- Translate upper arm relative to upper arm: \( T_{uu} \)
- Rotate upper arm around joint: \( R_{uu} \)
  - Apply \( M = R_b T_{lu} R_{lu} T_{uu} R_{uu} \) to upper arm

OpenGL Code for Robot

```c
mat4 ctm; // current transformation matrix
robot_arm()
{
  ctm = RotateY(theta);
  base();
  ctm *= Translate(0.0, h1, 0.0);
  ctm *= RotateZ(phi);
  lower_arm();
  ctm *= Translate(0.0, h2, 0.0);
  ctm *= RotateZ(psi);
  upper_arm();
}
```

Tree Model of Robot

- Note code shows relationships between parts of model
  - Can change "look" of parts easily without altering relationships
- Simple example of tree model
- Want a general node structure for nodes
Objectives

• Build a tree-structured model of a humanoid figure
• Examine various traversal strategies
• Build a generalized tree-model structure that is independent of the particular model

Building the Model

• Can build a simple implementation using quadrics: ellipsoids and cylinders
• Access parts through functions
  - torso()
  - left_upper_arm()
• Matrices describe position of node with respect to its parent
  - $M_{lla}$ positions left lower arm with respect to left upper arm

Humanoid Figure

Tree with Matrices

Possible Node Structure

Code for drawing part or pointer to drawing function

matrix relating node to parent

Generalizations

• Need to deal with multiple children
  - How do we represent a more general tree?
  - How do we traverse such a data structure?
• Animation
  - How to use dynamically?
  - Can we create and delete nodes during execution?
Display and Traversal

- The position of the figure is determined by 11 joint angles (two for the head and one for each other part)
- Display of the tree requires a graph traversal
  - Visit each node once
  - Display function at each node that describes the part associated with the node, applying the correct transformation matrix for position and orientation

Transformation Matrices

- There are 10 relevant matrices
  - \( M \) positions and orients entire figure through the torso which is the root node
  - \( M_h \) positions head with respect to torso
  - \( M_lua, M_rua, M_lul, M_rul \) position arms and legs with respect to torso
  - \( M_lla, M_rla, M_lll, M_rll \) position lower parts of limbs with respect to corresponding upper limbs

Stack-based Traversal

- Set model-view matrix to \( M \) and draw torso
- Set model-view matrix to \( MM_h \), and draw head
- For left-upper arm need \( MM_{lua} \), and so on
- Rather than recomputing \( MM_{lua} \) from scratch or using an inverse matrix, we can use the matrix stack to store \( M \) and other matrices as we traverse the tree

Traversal Code

```plaintext```
ffigure() {
 PushMatrix();
  torso();
  Rotate (...);
  head();
  PopMatrix();
  MM_h;
  PushMatrix();
  Translate(...);
  Rotate(...);
  left_upper_arm();
  PopMatrix();
  MM_lua;
  PushMatrix();
  ...
  ...
  rest of code
}
```

Analysis

- The code describes a particular tree and a particular traversal strategy
- Can we develop a more general approach?
- Note that the sample code does not include state changes, such as changes to colors
- May also want to use a PushAttrib and PopAttrib to protect against unexpected state changes affecting later parts of the code

General Tree Data Structure

- Need a data structure to represent tree and an algorithm to traverse the tree
- We will use a left-child right sibling structure
- Uses linked lists
- Each node in data structure is two pointers
- Left: linked list of children
- Right: next node (i.e. siblings)
**Left-Child Right-Sibling Tree**

![Diagram of Left-Child Right-Sibling Tree](image)

**Tree node Structure**

- At each node we need to store
  - Pointer to sibling
  - Pointer to child
  - Pointer to a function that draws the object represented by the node
  - Homogeneous coordinate matrix to multiply on the right of the current model-view matrix
    - Represents changes going from parent to node
    - In OpenGL this matrix is a 1D array storing matrix by columns

**C Definition of treenode**

```c
typedef struct treenode
{
    mat4 m;
    void (*f)();
    struct treenode *sibling;
    struct treenode *child;
} treenode;
```

**torso and head nodes**

```c
treenode torso_node, head_node, lua_node, ...;
torso_node.m = RotateY(theta[0]);
torso_node.f = torso;
torso_node.sibling = NULL;
torso_node.child = &head_node;

head_node.m = translate(0.0, TORSO_HEIGHT + 0.5*HEAD_HEIGHT, 0.0) * RotateX(theta[1]) * RotateY(theta[2]);
head_node.f = head;
head_node.sibling = &lua_node;
head_node.child = NULL;
```

**Notes**

- The position of figure is determined by 11 joint angles stored in `theta[11]`
- Animate by changing the angles and redisplaying
- We form the required matrices using `Rotate` and `Translate`
  - Because the matrix is formed using the model-view matrix, we may want to first push original model-view matrix on matrix stack

**Preorder Traversal**

```c
void traverse(treenode* root)
{
    if(root==NULL) return;
    mvstack.push(ctm);
    ctm = ctm*root->m;
    root->f();
    if(root->child!=NULL) traverse(root->child);
    ctm = mvstack.pop();
    if(root->sibling!=NULL) traverse(root->sibling);
}
```
Traversing Code & Matrices

```
figure() {
    PushMatrix();
    torso();
    Rotate(...);
    head();
   PopMatrix();
    PushMatrix();
    Translate(...);
    Rotate(...);
    left_upper_arm();
}
```

Notes

- We must save current transformation matrix before multiplying it by node matrix
  - Updated matrix applies to children of node but not to siblings which contain their own matrices
- The traversal program applies to any left-child right-sibling tree
  - The particular tree is encoded in the definition of the individual nodes
- The order of traversal matters because of possible state changes in the functions

Homework 5

- Create models for links (P0, P1, P2 & P3)
- Draw base model (P0)
  - $M = T_z(L0) \cdot R_y(\theta_1) \equiv \text{matrix multiply}$
  - Apply transformation matrix $M$ to first link model (P1): $P1' = M \cdot P1$
  - Draw $P1'$
  - $M = M \cdot T_z(L1) \cdot R_y(\theta_2)$
  - $P2' = M \cdot P2$ // Apply matrix to second link
  - Draw $P2'$

Homework 5 (cont.)

- $M = M \cdot (T_z(L2) \cdot R_y(\theta_3))$
- $P3' = M \cdot P3$ // Apply matrix to third link
- Draw $P3'$
- $M = M \cdot T_z(L3)$
- Extract translation vector from $M$ as the position for drawing sphere at end of arm