Models of the Bidirectional Reflectance Distribution Function

Steve Lombardi
Drexel University
Photo-realistic Image Synthesis
Image Formation
Surface Reflectance
Surface Reflectance

\[ \theta_o, \varphi_o \leftrightarrow \omega_o \text{ – Outgoing direction} \]

\[ \theta_i, \varphi_i \leftrightarrow \omega_i \text{ – Incident direction} \]

\[ f_r(\omega_i, \omega_o) = \frac{dL_o(\omega_o)}{L_i(\omega_i)(\mathbf{n} \cdot \omega_i)d\omega_i} \]  

[F. Nicodemus 65]
BRDF

\[ f_r(\omega_i, \omega_o) = \frac{dL_o(\omega_o)}{L_i(\omega_i)(\mathbf{n} \cdot \omega_i) d\omega_i} \]

Image Synthesis

**Given**
- \( f_r(\omega_i, \omega_o) \)
- \( L_i(\omega_i) \)
- \( \mathbf{n} \)

**Unknown**
- \( L_o(\omega_o) \)

Photometric Stereo

**Given**
- \( L_o(\omega_o) \)
- \( f_r(\omega_i, \omega_o) \)
- \( L_i(\omega_i) \)

**Unknown**
- \( \mathbf{n} \)
Physical Properties

- **Reciprocity**
  \[ f_r(\omega_i, \omega_o) = f_r(\omega_o, \omega_i) \]

- **Energy Conservation**
  \[ \int_{\Omega} f_r(\omega_i, \omega_o)(n \cdot \omega_o) d\omega_o \leq 1 \]

- **Positivity**
  \[ f_r(\omega_i, \omega_o) \geq 0 \]

- **Isotropy**
  \[ \forall k \quad f_r(\theta_i, \phi_i, \theta_o, \phi_o) = f_r(\theta_i, \phi_i + k, \theta_o, \phi_o + k) \]
  \[ f_r(\theta_i, \theta_o, |\phi_i - \phi_o|) \]
Desired Traits of a BRDF Model

- Compactness
  - Small number of parameters
- Expressiveness
  - Modeling a wide variety of behavior
- Editability
  - Ability to be intuitively modified
Modeling the BRDF

- Ideal models
- Physically-based models
- Non-parametric models
- Phenomenological models
Ideal Models :: Ideal Diffuse

\[ f_r(\theta_i, \phi_i, \theta_o, \phi_o) = \frac{\rho}{\pi} \]

- Scatters light equally in all viewing directions
- Known as Lambertian reflectance [J. Lambert 1760]
Ideal Models :: Ideal Specular

$$f_r(\theta_i, \phi_i, \theta_o, \phi_o) = \frac{\delta(\theta_i - \theta_o)\delta(\phi_i - \phi_o - \pi)}{\sin \theta_i \cos \theta_i}$$

- Perfectly smooth surface which reflects light
Ideal Models :: Ideal Retroreflector

\[ f_r(\theta_i, \phi_i, \theta_o, \phi_o) = \frac{\delta(\theta_i - \theta_o)\delta(\phi_i - \phi_o)}{\sin \theta_i \cos \theta_i} \]

- Light reflected back to incident direction
Microfacet Models

\[ n \]
Physically-based :: Torrance-Sparrow

- Specular microfacet model
  - Facets exhibit the Fresnel effect
- Facets distributed on surface probabilistically
- Only facets aligned to halfway vector contribute radiance
- Each facet has an adjacent opposing facet rotated 180° which form a “V-cavity”
  - Facets may mask each other or shadow incident light from one another
Physically-based :: Torrance-Sparrow

\[ f_r(\omega_i, \omega_o) = \frac{D(\theta_h)F(\omega_i, H)G(\omega_i, \omega_o, H)}{4 \cos \theta_i \cos \theta_o} \]

- \( D(\theta_a) \) • distribution of facet slopes
- \( F(\omega_i, H) \) • Fresnel term
- \( G(\omega_i, \omega_o, H) \) • geometric term

\[ D(\theta_a) = \frac{1}{\sqrt{2\pi\sigma}} \exp \left( -\frac{\theta_a^2}{2\sigma^2} \right) \]
Physically-based :: Torrance-Sparrow

Renderings of a sphere with the Torrance-Sparrow model with increasing surface roughness
Physically-based :: Torrance-Sparrow

- Advantages
  - Compact
  - Physically-based

- Disadvantages
  - Only models specular phenomenon
Physically-based :: Oren-Nayar

- Diffuse microfacet model
  - Lambertian facets
- Facets distributed on surface probabilistically
- All facets contribute to outgoing radiance
  - Requires integration
- Each facet has a partner rotated 180 degree which form a V-cavity
  - Facets may mask, shadow and interreflect

(a) Shadowing
(b) Masking
(c) Interreflection

[M. Oren 94]
Physically-based :: Oren-Nayar

(a) Image  (b) Lambertian  (c) Model
Physically-based :: Oren-Nayar

Wall Plaster

Sand Paper
Physically-based :: Oren-Nayar

- Advantages
  - Compact
  - Physically-based

- Disadvantages
  - Only model diffuse phenomenon
  - Only an approximation to the formulation
Physically-based Models

- Advantages
  - Compactness
  - Obeys physical BRDF constraints

- Disadvantages
  - Reduced expressiveness
Non-parametric Models

- Based on measured BRDF data
- Often high storage requirements
- Typically high accuracy and expressiveness
A Data-Driven Reflectance Model

- Introduces the MERL/MIT BRDF database
  - 100 measured isotropic BRDFs
- Linear and non-linear dimensionality reduction
  - Linear: 45 dimensions
  - Nonlinear: 15 dimensions
- User-defined description vectors
  - Enable intuitive editing
Non-parametric :: Data-Driven
Non-parametric :: Data-Driven

- Data is stored in a new parameterization
  - $\theta_i, \varphi_i, \theta_o, \varphi_o \leftrightarrow \theta_d, \varphi_d, \theta_h, \varphi_h$

- 90 samples of $\theta_d$, 90 of $\theta_h$, 180 of $\varphi_d$, 3 color channels
- 4,373,400 total samples
Non-parametric :: Data-Driven

- Each BRDF is 4,374,000-dimensional vector
- Using linear dimensionality reduction
  - Less than 1% reconstruction error with 45-dimensional subspace
  - Problem: dimensions do not always correspond to plausible BRDF

- Using non-linear dimensionality reduction
  - Charting dimensionality reduction algorithm
  - Steep reconstruction error drop-off with 15-dimensional space
Non-parametric :: Data-Driven

- Users categorize BRDFs according to certain properties
- Support vector machines find a classification hyperplane
- The normal of this hyperplane is used as a description vector
- Moving along this vector increases that characteristic
  - E.g. “redness”, “shininess”, “silverness”, etc.,
Non-parametric :: Data-Driven

Editing with “silverness” vector

Editing with “redness” vector
Non-parametric :: Data-Driven

- Advantages
  - Excellent way to edit BRDF
  - Captures a natural space of isotropic BRDFs
- Disadvantages
  - Difficult to generalize from only 100 samples
  - Extremely large vectors make it less useful for direct rendering
  - Description vectors are not orthogonal
    - Changing one trait might unintentionally affect another
Non-parametric :: Passive Reflectometry

- Motivated by difficulty of measuring BRDFs
- Utilizes parameterization as in previous model
  - $\theta_i, \varphi_i, \theta_o, \varphi_o \leftrightarrow \theta_d, \varphi_d, \theta_h, \varphi_h$
- Assuming isotropy reduces dimensionality to 3
- Assuming invariance along $\varphi_d$ reduces dimensionality to 2
- Non-linear sampling along $\theta_h$ for highlight precision
Nonparametric :: Passive Reflectometry
Non-parametric :: Passive Reflectometry

• Advantages
  • Represents a wide class of BRDFs with a low number of samples

• Disadvantages
  • Ignores additional data which may affect some materials
Non-parametric

- Advantages
  - Accurate and expressive

- Disadvantages
  - Requires large amounts of storage space
    - Not suited to real-time rendering
  - Unable to extrapolate/generalize
Phenomenological Models

- Attempt to represent BRDFs empirically
- Vary in expressiveness, compactness
- May not satisfy physical BRDF conditions
Phenomenological :: Lafortune

- Generalization of Phong’s reflectance model
  \[ f_r(\omega_i, \omega_o) = C_s (\omega_m \cdot \omega_o)^n \]
- Replaces dot product with weighted dot product
  \[ f_r(\omega_i, \omega_o) = (C_x \omega_{i,x} \omega_{o,x} + C_y \omega_{i,y} \omega_{o,y} + C_z \omega_{i,z} \omega_{o,z})^n \]
Phenomenological :: Lafortune

- $C_x = C_y = -1, C_z = 1$
  - Phong model
- $|C_x| = |C_y| > |C_z|$
  - Off-specular reflection
- $C_x = C_y > 0$
  - Retroreflective
- $C_x = C_y = 0$
  - Generalized diffuse
- $C_x \neq C_y$
  - Anisotropic
Phenomenological :: Lafortune
Phenomenological :: Lafortune

Laforutne (dashed lines) compared to HTSG (solid line)

Laforutne (dashed lines) compared to measured BRDF (solid line)
Phenomenological :: Lafortune

- Advantages
  - Compact
  - Fast

- Disadvantages
  - Cannot model all possible BRDFs
    - Work by Stark et al. shows this
Phenomenological :: Zernike Polynomials

- Set of orthogonal bases on the unit disk
- Can be transformed onto the hemisphere easily
- A function of the Cartesian product of the Zernike polynomials can be used as a basis function for the BRDF

Contour density plots of the Zernike polynomials

[J. Koenderink 98]
Phenomenological :: Zernike Polynomials

- BRDF as linear combination of basis functions
  \[ f_r(\theta_i, \phi_i, \theta_o, \phi_o) = \sum_{mnkl} a_{nm} H_{nm}^{kl}(\theta_i, \phi_i, \theta_o, \phi_o) \]

- Each basis function is a combination of Zernike polynomials K transformed onto the hemisphere
  \[ H_{nm}^{kl}(\theta_i, \phi_i, \theta_o, \phi_o) \propto K_n^k(\theta_i, \phi_i) K_m^l(\theta_o, \phi_o) + K_n^k(\theta_o, \phi_o) K_m^l(\theta_i, \phi_i) \]

- Automatically respects reciprocity
- 295 basis functions for total order 8
  - Much too large
Phenomenological :: Zernike Polynomials

- Isotropic scattering only

\[ f_r(\theta_i, \phi_i, \theta_o, \phi_o) = \sum_{nml} a_{nm}^l I_{nm}^l(\theta_i, \phi_i, \theta_o, \phi_o) \]

- Only based on azimuthal difference

\[ I_{nm}^l(\theta_i, \phi_i, \theta_o, \phi_o) \propto K_n^l(\theta_i, |\phi_i - \phi_o|)K_m^l(\theta_o, |\phi_i - \phi_o|) + K_m^l(\theta_i, |\phi_i - \phi_o|)K_n^l(\theta_o, |\phi_i - \phi_o|) \]

- 55 basis function of total order 8
Phenomenological :: Zernike Polynomials

- **Advantages:**
  - Graceful compression by truncation
  - Easy fitting by convolution

- **Disadvantages**
  - Large dimensionality for accurate representation
  - Ringing side-effects from truncation
Phenomenological :: Halfway Vector Disk

- Designed for Monte Carlo rendering
  - Uses probabilistic formulation of BRDF
- Samples halfway vector instead of incident direction
- Samples halfway vector by sampling from halfway vector disk
Phenomenological :: Halfway Vector Disk

- “Lump” PDFs on halfway vector disk
  \[ p(h) \propto \left[ 1 - \frac{||h - c||^2}{R^2} \right]^n \]

- Two BRDF models proposed with PDFs
  - Empirical model
    - Radii confined within unit disk
    - One specular lump, one retroreflective lump
  - Data-fitting model
    - Radii not confined to unit disk
    - Two specular lumps
Phenomenological :: Halfway Vector Disk

Reference image rendered with measured data

Rendered with data-fitting model

Rendered with empirical model
Phenomenological :: Halfway Vector Disk

Rendered with data-fitting model

Rendered with Lawrence et al. BRDF
Phenomenological :: Halfway Vector Disk

- **Advantages**
  - Superior performance for Monte Carlo rendering
  - Empirical model perfectly conserves energy
- **Disadvantages**
  - Does not enforce reciprocity
# Model Comparison :: Physical Properties

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<th></th>
<th>Reciprocity</th>
<th>Energy Conservation</th>
<th>Positivity</th>
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<tbody>
<tr>
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<td>Lafortune*</td>
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Lafortune model can be positive *under* the hemisphere.*
## Model Comparison :: Desired Traits

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<th>Intuitive editing</th>
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Model Comparison :: Rendered Scenes

- Measured
- Lambertian + Torrance-Sparrow
- Lambertian + Lafortune
Conclusion

- Applications drive model characteristics
  - Excellent for fast, expressive rendering: Lafortune
  - Outstanding for editing: Data-Driven Reflectance Model

- Alternate parameterization is key
  - Halfway-based formulations excel
  - Finding natural variation among reflectance parameters
References


References


