Questions from Last Time?
- Sampling Theory
- Fourier Analysis
- Anti-aliasing
- Supersampling Strategies

Slide Credits
- Charles B. Owen - Michigan State University

Illumination and Shading
- The Hall illumination model
- Original ray tracing paper

Discrete Illumination Models
- What occurs when light strikes a surface is quite complex.
  - Continuous process
  - Light from infinite angle reflected in infinite directions
- We are determining intensity of a pixel with...
  - Finite number of point lights
  - Finite reflections into space
  - Finite illumination directions
- Hence, we must have a discrete model for lighting and illumination.
Elements of Lighting at a point

- N – The surface normal
- L – Vector to the light
- V – Vector to the eye
- R – Reflection direction

Reflection

- What we need is the amount of light reflected to the eye

This consists of several components…

Diffuse Reflection

- Diffuse reflection - light reflected in all directions equally (or close to equally)
  - Most objects have a component of diffuse reflection
    - other than pure specular reflection objects like mirrors
  - What determines the intensity of diffuse reflection?

Diffuse Reflection Characteristics

- Since the intensity is the same in every direction, the only other characteristic is the angle between the light and the surface normal. The smaller this angle, the greater the diffuse reflection:

Lambert’s Law

\[ \cos \theta = \frac{w}{w'} \]

\[ w = w' \cos \theta \]

Diffuse reflection decreases intensity by the cosine of the angle between the light and surface normal.

Specular Reflection

- Specular reflection - If the light hits the surface and is reflected off mostly in a reflection direction, we have specular reflection.
  - There is usually some diffusion.
  - A perfect specular object (no diffusion at all) is a mirror.
  - Most objects have some specular characteristics.
**Diffuse and Specular colors**

- Typically the colors reflected for diffuse and specular reflection are different
  - Diffuse – Generally the surface appearance
  - Specular – The color of bright highlights, *often more white* than the surface color

**Where do these come from?**

- Most surfaces tend to have:
  - Deep color, the color of the paint, finish, interior material, etc.
  - Diffuse Color
  - Surface reflection characteristics, varnish, polish, smoothness
  - Specular Color

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**The Hall Illumination Model**

\[
I(\lambda) = k_s \sum I_s(\lambda) F_s(\lambda, \theta_s)(\cos \theta_s)^m + k_d \sum I_d(\lambda) F_d(\lambda, \theta_d)(\cos \theta_d)^n + k_t \sum I_t(\lambda) F_t(\lambda, \theta_t) + k_a \sum I_a(\lambda) F_a(\lambda)
\]

- Specular Reflection from Light Sources
- Specular Transmission from Light Sources
- Diffuse Reflection from Light Sources
- Specular Reflection from other surfaces
- Specular Transmission from other surfaces
- Ambient Light

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**Hall Model**

- Implementing Shadows
  - At every ray intersection shoot a “shadow” ray at every point light source in your scene
  - If an object is hit before reaching the light, don’t use the light when shading that intersection point
  - What if occluding object is “transparent”?
    - Make light’s contribution weighted rather than binary
**Ambient Light**

- Ambient light is light with no associated direction. The term in the Hall shading model for ambient light is:
  \[ k_{dr} I_a(\lambda) F_{dr}(\lambda) \]

- \( k_{dr} \) is the coefficient of diffuse reflection.
  - This term determines how much diffuse reflection a surface has. It ranges from 0 to 1 (as do most of these coefficients).

**Implementation**

- It’s common to combine \( k_{dr} \) and \( F_{dr}(\lambda) \)
  - \( F_{dr}(\lambda) \) is really just a (diffuse) color
  - \( I_a(\lambda) \) is the ambient light color
  - Can just define an ambient surface color
  - Possible implementation
    - for (int c=0; c<3; c++)
      - hallcolor[c] = lightambient[c] * \( k_{dr} \) * surfacediffuse[c];

**Diffuse Reflection of Light Sources**

- The iterator \( j \) takes on the index of every light in the system.
  - \( k_{dr} \) - coefficient of diffuse reflection.
  - \( I_s(\lambda) \) - spectrum of light source \( j \).
    - It is simply the color of the light.
  - Can combine \( k_{dr} \) and \( F_{dr}(\lambda) \) to produce a single diffuse surface color.
  - \( N \cdot L_j \) component.
    - \( N \) is the surface normal at the point.
    - \( L_j \) is a vector towards the light.
    - Dot product is the cosine of the angle (vectors must be normalized).
    - Value decreases as the angle increases.
Specular Reflection of Light Sources

\[ k_s \sum I(\lambda) F_s(\lambda, \theta_r, j)(\cos \theta_r, j)^n \]

- \( k_s \) and \( l_s(\lambda) \) are now obvious.
- \( F_s(\lambda, \theta_r, j) \) is the Fresnel term representing the specular reflection curve of the surface.
  - Specular reflection is due to microfacets in the surface and this curve can be complex. In real world systems which strive for accuracy, this curve will be measured for a given material. Note that the curve is dependent on not only the wavelength, but also an angle.
  - A simplification of this is to ignore the angle, which is what we will do.
  - But, the color of spectral highlights is independent of the color of the surface and is often just white.

The Spectral Intensity Function

\[ k_s \sum I(\lambda) F_s(\lambda, \theta_r, j)(\cos \theta_r, j)^n \]

- \( (\cos \theta_r, j)^n \) is the spectral intensity function.
  - It represents the cosine of the angle between the "half" reflection vector and the surface normal raised to a power.
  - Maximum reflection is in the "mirror" direction.

Cosine of Reflection Angle

\[ k_s \sum I(\lambda) F_s(\lambda, \theta_r, j)(\cos \theta_r, j)^n \]

\[ \cos \theta_r, j = N \cdot H \]

\[ H = \frac{L + V}{|L + V|} \]

H bisects the angle between L and V

Reflection Angles

This is an example of maximum reflection.

In this case, the "half" vector is the same as the surface normal.

Cosine of angle between half vector and normal is 1.

Different Than Phong Shading

\[ \cos \theta_r, j = N \cdot H \]

Hall Model

\[ \cos \phi_r, j = V \cdot F_j \]

Phong Model
Hall vs. Phong Specular

- Same when V = R
  - \( \cos(\theta) = \cos(\phi) = 1 \)
- Phong goes to zero when V \perp R. Then it goes negative!
- Hall only goes to zero when L = V \&\& \perp N.
- Hall never negative.

Specular Reflection Highlight Exponent

- The term \( n \) is called the specular reflection highlight exponent.
- This affects how large the spectral highlight is. A larger value makes the highlight smaller and sharper.
  - Matte surfaces have smaller \( n \).
  - Very shiny surfaces have large \( n \).
  - A perfect mirror would have infinite \( n \).

Specular Reflection from Other Surfaces

\[ k_{sr} I_{sr}(l) F_{sr}(\lambda, \theta_R) T_{sr} \]

- This is reflections of other surfaces
- The only new terms are \( I_{sr}(l) \) and \( T_{sr} \)
  - The \( T_{sr} \) term reflects the fact that light falls off exponentially with distance. \( T_r \) is a term which models how much light falls off per unit of travel within the medium.
  - The \( \Delta sr \) term represents how far the light travels. Note that for mediums such as air and a small scene \( T_r \) is close to 1, so you can sometimes ignore it.
  - Glassner suggests \( 1/(1 + a \cdot \Delta sr) \) instead

The Reflection Direction

- Given a view direction V and a normal N, the reflection direction R is:
  \[ R = 2(N \cdot V)N - V \]
- \( I_i(\lambda) \) is the color seen in the reflection direction, i.e. the color returned from the reflection ray.
- \( k_{sr} \) and \( F_{sr}(\lambda, \theta_{t,j}) \) can be combined to create a single “reflection surface color”.

Transmission

- Transmission is light that passes through materials

Specular Transmission from Lights

\[ k_{sr} \sum_{j} I_j(\lambda) F_{sr}(\lambda, \theta_{t,j}) (\cos\theta_{t,j})^n' \]

- Bright spots from lights passing through objects. Most of the same issues apply.
- \( I_j(\lambda) \) is the color of light \( j \).
- \( F_{sr}(\lambda, \theta_{t,j}) \) specular transmissive color of object.
- \( (\cos\theta_{t,j})^n' \) is how the specularity falls off if looking directly down the direction of transmission.
- Can be omitted from your shading.
Refractive Transmission

- Given indices of refraction on above and below a surface, we can compute the angle for the view and transmission vectors using Snell’s law.

\[
\sin \theta_i = \frac{\eta_j}{\eta_i} \\
\sin \theta_j = \frac{\eta_i}{\eta_j}
\]

\(\eta_i\) and \(\eta_j\) are the indices of refraction for the front and to objects respectively.

What Transmission Looks Like

\[
\frac{V - \beta L}{\beta - 1}
\]

\(\cos \theta\)  \(\text{Max time is } (-N \cdot H')^T\)

Index of Refraction

- Ratio of speed of light in a vacuum to the speed of light in a substance.

<table>
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<tr>
<th>Substance</th>
<th>Index</th>
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<tr>
<td>Vacuum</td>
<td>1.0</td>
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<td>Air</td>
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<td>Sapphire</td>
<td>1.77</td>
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<tr>
<td>Salt</td>
<td>1.54</td>
</tr>
</tbody>
</table>

The Transmission Direction

\[
T = \left( \eta_i (N \cdot V) - \sqrt{1 - \eta_i^2 (1 - (N \cdot V)^2)} \right) N - \eta_i V
\]

See Glassner p. 134-141

Total Internal Reflection

- If light is traveling from \(\eta_i\) to a smaller \(\eta_j\) (e.g. out of water into air), the angle from the negative normal increases:

This can lead to the angle for \(T\) being \(\geq 90\) degrees!

This is called total internal reflection.

Square root term in previous equation goes negative.

Total Internal Reflection Hack

What to do if square root is negative?

- Return object’s reflective color, or
- Shoot an internal reflective ray, or
- But, do what at bottom of ray tree?
  - Return transmissive color?
  - Make \(T\) equal to \(-V\)
  - Do the one that looks “best”
Specular Transmission from Other Surfaces

- This is transmitted light from other objects.
- $k_{st}$ - specular transmission coefficient.
- $I_{st}(\lambda)$ is the color value returned by transmitted ray.
- $F_{st}(\lambda, \theta_T)$ specular transmissive color of object.
- $T_{st}$ is the light falloff term, as before.
- Glassner suggests $1/(1 + a \cdot D_{st})$ instead.
- $\Delta_{st}$ is distance traveled through object.

\[
I(\lambda) = k_r \sum I_{sr}(\lambda) F_{sr}(\lambda, \theta_R) + k_a \sum I_{sr}(\lambda) F_{sr}(\lambda, \theta_R) T_{sr} + k_{st} I_{st}(\lambda) F_{st}(\lambda, \theta_T) T_{st} + k_{dr} I_{a}(\lambda) F_{dr}(\lambda, N \cdot L)
\]

Hall Model Recap

- For every primary ray that hits object:
  - Shoot shadow ray at each light.
  - If you hit something on the way, don’t use light.
  - Or decrease light color if object is transparent.
- Calculate diffuse, specular and ambient colors.
- Shoot reflected ray. Calculate color $I_{sr}(\lambda)$ and add it to the shading equation.
- Shoot transmitted ray. Calculate color $I_{st}(\lambda)$ and add it to the shading equation.
- Calculate transmissive specular highlight, if light ray goes through transparent object.

Algorithm Overview

- Adding two new coefficients in order to provide more flexibility to the model.
- $k_r$ - Reflection coefficient.
  - If set to zero, the object is not reflective.
  - This will allow you to generate Phong-shaded models.
- $k_a$ - Ambient coefficient.
  - Allows you to set ambient properties separate from diffuse properties.

New Parameters

- Generating trees for moderately complex scenes.
  - Light is reflected from several surfaces before reaching the viewer.
  - The light ray behavior can be modeled as a tree.
  - The shader traverses the tree, applying the shading equation at each node to calculate intensity.
  - Tree should have a preset maximum depth.

Tricks

- Terminate shadow ray ASAP.
- Limit the depth of ray tree.
- Recursion can be stopped when color value difference drops below $e$.
- Shooting a secondary ray can be tricky.
  - Move start point slightly away from surface in the direction of the local surface normal.
- What if shadow ray hits a non-opaque object?
- What to do with total internal reflection and coming to the bottom of ray tree?
Next Programming Assignments

- Update shading equation
  - New specular angle
  - k's can be combined with C's, e.g.,
    - \( C_r = k_r \cdot C_s \)
    - \( C_d = k_d \cdot C_d \)
- Add shadows (A5)
- Add reflections (A6)

\[
I = k_s \sum_j I_{ij} C_s (\cos \phi) \]
\[
+ k_a \sum_j I_{ij} C_d (\cos \theta) \]
\[
+ k_r I_r C_s \]
\[
+ k_a I_a C_d \]