Hair Simulation Model for Real-Time Environments

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Overview

• What are they trying to achieve?
• How was it done previously?
• What is the new approach?
• Conclusion & Future enhancements
What?

- Animating hair in real-time scenarios
Previous Research

- Volume Based
- Hair treated as a volume of "hair matter"
- generally Faster but cannot capture complex hair behavior
Previous Research

• Strand Based

• Some Frameworks exist, but cannot compute in real-time
New Approach?

• Elastic Rod based

• collection of individual leader strands (subject to physical simulation)

• greater number of follower strands (interpolated from leaders)

• advantage: keeps simulated strands at a manageable level, allows non-uniform behavior
New Approach: Details

• Characteristics of Hair
• Algorithm Overview
• Characteristics of Elastic Rods
• Model Discretization
• Energy and Motion Calculations
Characteristics of Hair

- hair bends & twists
- typical human >100,000 hair strands
- unstretchable & unshearable
- length of typical hair strand is longer than its diameter
Algorithm Overview

calculate initial values

do simulation
  calculate elastic forces
  calculate motion
  detect hair-head collisions
end
Characteristics of Elastic rods

- deformable body
- one dimension is significantly larger than its cross section
- Inextensible & unshearable
- Elastic energy (Kirchhoff theory of elastic rods) = bending + twisting
Elastic Energy

- The bending and twisting energies are described by the following equations:

\[ E_{\text{bend}} = \frac{1}{2} \int_0^L (\omega(s) - \hat{\omega}(s))^T \hat{B}(s)(\omega(s) - \hat{\omega}(s)) ds \]

\[ E_{\text{twist}} = \frac{1}{2} \int_0^L \beta(s)(m(s) - \hat{m}(s))^2 ds \]
Model Discretization

- Each hair strand is discretized into \( n+2 \) nodes connected by \( n+1 \) segments

\[
\begin{align*}
\text{nodes } & x_0, \ldots, x_{n+1} \\
\text{segments } & e^0, \ldots, e^n
\end{align*}
\]
Model Discretization

- Hair (generally) has elliptical cross-section
Model Discretization

- Bending Energy discretization
- Twist Energy discretization

\[ E_{\text{twist}} = \frac{1}{2} \int_0^L \beta(s)(m(s) - \hat{m}(s))^2 \, ds \]

\[ E_{\text{twist}}(\Gamma) = \sum_{i=1}^n \beta \frac{(m_i - \hat{m}_i)^2}{l_i} \]

\[ E_{\text{bend}} = \frac{1}{2} \int_0^L (\omega(s) - \hat{\omega}(s))^T \hat{B}(s)(\omega(s) - \hat{\omega}(s)) \, ds \]

\[ E_{\text{bend}}(\Gamma) = \sum_{i=1}^n \frac{1}{2l_i} \sum_{j=i-1}^i (\omega_i^j - \hat{\omega}_i^j)^T \hat{B}^j (\omega_i^j - \hat{\omega}_i^j) \]

\[ B^j = \begin{pmatrix} \mu \alpha^j & 0 \\ 0 & \alpha^j \end{pmatrix} \]
Calculate Motion

• Motion for rod's dynamic behavior (Mass $\times$ Acceleration = Force)

• Elastic force: strives to minimize the elastic energy

• External Force: gravity + friction against static ambient air

\[ M_i \ddot{x}_i = F_i^{\text{elastic}}(x) + F_i^{\text{external}}(x, \dot{x}) \]

\[ F_i^{\text{elastic}} = -\frac{dE(x)}{dx_i} \]

\[ F_i^{\text{external}} = M_i g - v_i \dot{x}_i \]
Results

• Scenario #1
  • interactive setup, several wisps of hair attached to a stand.
Results

- Scenario #2
- full head of hear, 50 leader strands, 5,000 follower strands.
Results

- Performance
- Intel Core 2 Quad 2.66 Ghz, 8 GB RAM

<table>
<thead>
<tr>
<th>Scenario</th>
<th>#leaders</th>
<th>#strands</th>
<th>#nodes</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stand #1 long</td>
<td>3</td>
<td>72</td>
<td>90</td>
<td>1.10</td>
</tr>
<tr>
<td>Stand #2 short</td>
<td>3</td>
<td>72</td>
<td>15</td>
<td>0.13</td>
</tr>
<tr>
<td>Stand #3 short</td>
<td>18</td>
<td>450</td>
<td>90</td>
<td>0.52</td>
</tr>
<tr>
<td>Head #1 straight</td>
<td>50</td>
<td>5000</td>
<td>1500</td>
<td>24.49</td>
</tr>
<tr>
<td>Head #2 straight</td>
<td>25</td>
<td>2500</td>
<td>750</td>
<td>12.93</td>
</tr>
<tr>
<td>Head #3 wavy</td>
<td>25</td>
<td>2500</td>
<td>750</td>
<td>13.27</td>
</tr>
</tbody>
</table>

Table 1: Test scenario performance; time in milliseconds
Results

- Hair-head collision performance

<table>
<thead>
<tr>
<th>Scenario</th>
<th>#leaders</th>
<th>#nodes</th>
<th>Collisions?</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Head #1 straight</td>
<td>50</td>
<td>1500</td>
<td>Yes</td>
<td>14.35</td>
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<tr>
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<td></td>
<td></td>
<td>No</td>
<td>13.54</td>
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<tr>
<td>Head #2 straight</td>
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<td>750</td>
<td>Yes</td>
<td>7.87</td>
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<td></td>
<td>No</td>
<td>7.03</td>
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<tr>
<td>Head #3 wavy</td>
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<td>750</td>
<td>Yes</td>
<td>7.97</td>
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<tr>
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<td></td>
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<td>No</td>
<td>7.08</td>
</tr>
</tbody>
</table>

Table 3: Performance impact of hair-head collision treatment, constraint enforcement time given in milliseconds
Conclusion

• Algo for dynamic simulation of hair in a real-time environment

• handled hair-head collisions
Future

- Integration with haptic device, 3D user interface
- Implement on full programmable GPUs exploiting the parallel nature of the model.