Ordering and Parameterizing Scattered 3D Data for B-Spline Surface Approximation

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Introduction

• Goal: construct a $C^n$-continuous B-Spline surface without a patch network.
• Extended Gaussian map has been used for both ordering and parameterizing.
• Estimating the control points based on MMSE (Minimum Mean Square Error).
• Solve the problem of many-to-one mapping of the extended Gaussian map.
B-Spline Surface Representation

• The B-Spline surfaces are piecewise polynomials or rational functions in two topological directions that describe the surface shape through a parametric representation using a relatively small number of control points.

• A B-Spline surface consists of continuous surface patches that are continuously connected at their boundaries and have continuous higher order derivatives.

• For example, a cubic B-Spline is continuous and has continuous tangents and curvatures.
Attractive Properties of B-Splines

• A B-Spline possesses a high degree of continuity important for computing the surface intrinsic properties, e.g., curvature.

• Affine invariance: a B-Spline subjected to an affine transformation is still a B-Spline whose control points are obtained by subjecting the original B-Spline control points to that affine transformation.

• Local shape controllability: Due to the local support of the basis B-Spline function, any local deformation is locally confined. This is very important when trying to register objects in the presence of missing parts.
B-Splines Surface Structure

\[
\mathbf{r}(u, v) = \frac{\sum_{i=0}^{m} \sum_{j=0}^{n} C_{ij} N_{i,4}(u) N_{j,4}(v) w_{ij}}{\sum_{k=0}^{m} \sum_{l=0}^{n} N_{k,4}(u) N_{l,4}(v) w_{kl}} = \sum_{i=0}^{m} \sum_{j=0}^{n} C_{ij} R_{i,j}(u, v)
\]

- mxn control points \( C_{ij} \)
- \( N_{i,4} \) and \( N_{j,4} \) are basis functions
- \( u, v \) parameter
- when \( \omega = 1 \), NURBS -> B-Spline
Meaningful Parameterization*

- 2500-point human face
- Scattered Data
Meaningful Parameterization*

- Divide x into \( \Delta x \) bins.
- Assign \( u \) value to the points in each bin.
- Divide y into \( \Delta y \) bins.
- Assign \( v \) value to the points in each bin.

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Meaningful Parameterization*

- Previous algorithm does not work on this case.
- Solution: Project onto Gaussian Sphere
Meaningful Parameterization*

\[ \phi = \cos^{-1}\left(\frac{z}{\sqrt{x^2 + y^2 + z^2}}\right), \theta = \tan^{-1}\left(\frac{y}{x}\right), 0 \leq \theta \leq \pi, 0 \leq \theta \leq 2\pi. \]

- A surface data point \( r \cdot x; y; z \) is ordered according to where it is projected onto the unit Gaussian sphere, i.e., in terms of its \( \phi; \theta \) values given by
Meaningful Parameterization*

- Divide $\theta$ into $\Delta \theta$ bins.
- Assign $u$ value to the points in each bin.
- Divide $\phi$ into $\Delta \phi$ bins.
- Assign $v$ value to the points in each bin.
Estimating the B-Splines Parameters

\[ \min \sum_{k=0}^{K} \left| \left| r_k - r(u_k, v_k) \right| \right|^2 \]

1. Set \( w_{ij} = 1 \) for all \( i, j \), and estimate the \( C_{ij} \)'s by minimizing the error in (3.1). The error in (3.1) is a quadratic function in the \( C_{ij} \)'s, hence, a closed-form solution is obtained;

2. Using the estimates in step 1 perform an iterative minimization of the error in (3.1), but now with respect to the \( w_{ij} \)'s. Using the new \( w_{ij} \) values, check whether the error in (3.1) is less than a predetermined value \( \varepsilon \). If yes, then terminate, otherwise go to step 3;

3. Using the estimated \( w_{ij} \)'s in step 2, minimize (3.1) w.r.t. the \( C_{ij} \)'s. Here again, a closed-form solution for the \( C_{ij} \)'s is obtained. Using the new \( C_{ij} \) values, check whether the error in (3.1) is less than \( \varepsilon \). If yes, terminate. Otherwise go to step 2. We iterate between step 2 and step 3 until convergence, i.e., until the error in (3.1) is less than \( \varepsilon \).
Experimental Results

TABLE 1
Average, Standard Deviation, and Maximum Value of the Residual Error Associated with B-Spline and NURBS Fit

<table>
<thead>
<tr>
<th>Surface</th>
<th>B-Splines</th>
<th></th>
<th>NURBS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Average</td>
<td>STD</td>
<td>Average</td>
</tr>
<tr>
<td>Head</td>
<td>0.0039</td>
<td>0.0035</td>
<td>0.0351</td>
</tr>
<tr>
<td>Local-deformed head</td>
<td>0.0049</td>
<td>0.0052</td>
<td>0.0650</td>
</tr>
<tr>
<td>Noise contaminated head</td>
<td>0.0060</td>
<td>0.0041</td>
<td>0.0389</td>
</tr>
<tr>
<td>3D Sinc Surface</td>
<td>0.4508</td>
<td>0.3523</td>
<td>1.4633</td>
</tr>
</tbody>
</table>

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Conclusions and Discussions

• It has difficulties in finding an ordering and a choice for the topological parameters of the B-Spline that lead to a physically meaningful surface parameterization based on the scattered data set.

• To achieve such a parameterization, we made use of the surface extended Gaussian map.
Thank you!
Any questions?