A Particle-Based Model for Simulating the Draping Behavior of Woven Cloth

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Abstract

We demonstrate a physically-based technique for producing draping simulations of a variety of woven fabrics. Our approach employs an interacting-particle model which is based on the microstructure of woven cloth, rather than utilizing a continuum approximation. Empirical data from a fabric testing device is used to tune energy functions within the model. We describe the model, how we convert the fabric test data to energy functions, and two experiments conducted to evaluate the approach. The first experiment produces non-linear mechanical data from the model. The second experiment compares photographs of three different types of draping cloth with visualizations of simulation results. The experiments show that we are able to reliably recover quantitative mechanical information from the model, and to reproduce the unique large-scale draping characteristics of a range of fabric types.
1 Introduction

The drape of woven materials has intrigued humans for centuries. This is evident in the flowing robes contained in the sculptures of ancient Rome, the intricate folds of fabrics depicted in the paintings of the Renaissance, and the elaborate billowing clothing of the 18th century. Even in modern times artists such as Christo realize that the image of draping cloth over a structure like the Reichstag in Berlin is fascinating and provocative. It has always been clear that woven materials have unique properties that allow them to deform in ways significantly different than other sheet materials, e.g. paper, vinyl, and metal foils. Cloth’s special deformation capabilities have been noted and recognized through the ages, but never really fully understood from a scientific or engineering perspective.

In engineering, great progress has been made in developing theories that explain and predict the deformation behavior of nearly-rigid materials like steel and stiff plastics. This has made possible the development of robust tools for computer-aided design (CAD), allowing engineers to design and analyze steel structures on computers, long before any real structure is actually built. Unfortunately, the same is not true for flexible materials such as cloth. For these kinds of materials few CAD tools exist, forcing apparel designers to use more traditional and less efficient design methods. One of the main reasons for the dearth of apparel CAD tools is the lack of good mechanical models of fabric that may be used to simulate and predict the folds and buckles of a draping cloth.

In this paper we report on a new technique for reliably reproducing the characteristic draping behavior of particular fabrics [1]. It attempts to answer questions like “What would this shirt look like made from cotton instead of polyester?” or “Would this dress have a more pleasing drape if made from silk rather than a light wool?” Our work on this problem began over four years ago with the development of a new theoretical model of woven cloth that is based on interacting-particle methods. Our most recent efforts, and those that form the focus of this paper, have been to develop techniques for using test data from the well-known Kawabata Evaluation System fabric measuring equipment [32] to tune the model. With these techniques we can now test a particular cloth sample, and then use the model to produce a draping behavior that is characteristic of this type of fabric. To verify the validity of our approach, we present results from experiments demonstrating that the modeled material behaves like the actual material, both in measured mechanical characteristics and in reproducing characteristic large-scale draping behavior.

To date, most of the efforts to create a model of cloth have employed continuum mechanics, with simulations utilizing finite element or finite difference techniques. In these approaches a deformable material can be modeled in two ways. In the first, the macroscopic behavior of the material is observed, continuous constitutive equations that describe the behavior are derived, and these equations are applied to small elements of a mesh used to represent the geometry of the material. This approach assumes that the small-scale behavior of a material is simply a scaled-down version of its macroscopic behavior. Another continuum approach recognizes the low-level structures of the material, but makes assumptions about the statistical distribution of these structures and mathematically aggregates their properties into continuous differential equations. These statistically derived equations are then used to define the properties of the elements of the modeled material. The details of the small-scale mechanical interactions within the material are lost in the process.

Cloth, on the other hand, has defied attempts by textile scientists to create an accurate model using continuum techniques. The problem is that the assumptions on which these techniques are based, that the behavior of a small element is governed by the same continuum equations that describe the large-scale behavior, are not valid for woven materials. In fact, cloth is a complex mechanism whose important mechanical elements are at a scale that is not far from the scale of the mesh element that would be used in a typical simulation. In cloth, fine fibers are spun into yarns or threads, and these threads are more or less tightly woven into an interlocking network. The complexity of the resulting mechanical system is clearly indicated by the microscopic view of a piece of wool cloth shown in Figure 1.1. Significantly, all of these components are held together not by molecular bonds or welds, but simply by friction. Behavior depends on everything from the type of fiber (e.g., silk, cotton, polyester, etc.), to the weight of the yarn, to the tightness and type of the weave.

The opinion that continuum models, appropriate for modeling steel beams and elastic membranes, cannot accurately model the limp, highly deformable, anisotropic, and non-linear behavior of woven materials, was first advanced by Shanahan, Lloyd and Hearle in 1978 [46]. In the introduction to a major study on
textile mechanics, they stated

"Because of the relative coarse structure of textile materials, . . . it might be more profitable . . . to use noncontinuum systems directly in the problems of complex (fabric) deformation."

Even though they had reservations about the applicability of continuum models to the problem of fabric drape, they explored them for many years, probably because the computational resources needed to develop a microstructural model of cloth were not readily available in the late 1970's. Shanahan et al. considered such an effort, but dismissed it as impractical.

"In studying the complex deformations of textile fabrics, such as occur in draping, it is not likely that it would be practical to model a fabric in detail as an assemblage of its constituent fibers or yarns."

It is noteworthy that after years of attempts to develop a satisfactory model using continuum techniques, Hearle finally abandoned this approach, stating [1]

"In dealing with 3-dimensional buckling of textile fabrics, neither the terminology nor the methodology of established (continuum) theory of bending plates and shells is of much help."

Our experience is that it is now both possible and practical to construct a model that captures key elements of the small-scale structure of woven cloth. Our model is an application of interacting-particle techniques. In contrast to continuum techniques, this approach attempts to capture low-level microscopic interactions within a material, and is founded on the premise that by modeling the low-level structures of a material and computationally aggregating their interactions, correct macroscopic behavior will emerge.

In the following sections we first present some key background material. We briefly review previous work in both particle systems and cloth modeling. Next, we present a summary of the fundamental elements of our woven cloth model. The remainder of the paper details our technique for incorporating empirical mechanical data into the theoretical model, and describes the experiments used to verify our approach.

2 Background

2.1 Particle Systems

Particle Systems were first presented to the computer graphics community by Reeves in 1983 [43] when he described the technique used to create the fire that sweeps over a planet in the Genesis Sequence of
the movie *Star Trek II: The Wrath of Khan*. He defined a particle system model as “a cloud of primitive particles,” where each particle is generated into the system, moves, changes form and color, and then dies. Reeves and Blau [44] present the concept of structured particle systems to create trees and grass. Others have utilized particle systems to simulate a variety of physical phenomena [10, 26]. Sims [50] demonstrated how particle systems could be efficiently implemented on a massively parallel computer to produce stunning visual effects of waterfalls, vortex fields, fire, snowstorms and explosions.

In this early work the particles interacted only with their environment, but not with each other. Reynolds [45] presents the idea of a coupled particle system, a system where the particles interact with each other, in his work to model the flocking and herding of animals. He did not use particle system terminology, but he did present the concept of defining simple local interactions between large numbers of small independent actors to produce complex aggregate behaviors. Miller and Pearce [39], Terzopoulos, Platt and Fleischer [56], and Tonnese [59] all explored coupled particle systems as a way to model liquid-like and melting materials. The interactions of the particles in these systems were independent of direction, producing spherical “blobby-like” objects. Haumann et al. [23, 24], Miller et al. [40], Szeliski and Tonnese [52], and van Wijk [60] proposed particle interactions that are a function of direction, producing deformable sheets and surfaces of particles. Our interest in coupled particle systems began with an exploratory study of particle systems as modeling primitives for a surgical simulator [27]. This led us to develop particle-based modeling, rendering, and simulation tools [7, 25, 58]. We have also explored the computational aspects of particle system models [28, 29, 41]. Our current work in this area involves applying particle systems ideas to the cloth modeling work described in this paper, and to the problem of developing CAD technologies for the design of broadcloth composite parts [3, 4, 5]. Our efforts to develop a computer model of cloth were originally part of a larger project that explored the automated handling of garments [33].

### 2.2 Cloth Modeling

Efforts to model the complex three-dimensional buckles and folds produced by cloth have been conducted by two distinct research communities, each working towards different goals [12]. In the computer graphics community, there has been significant work in the area of physically-based models of deformable sheets. These models have been developed mostly for use within computer animation. Thus, the focus has been on creating computationally-tractable models, based on physical principles, that produce “cloth-like” behavior. There has been no emphasis on producing accurate simulations of any particular material. The other body of work comes out of the engineering and Computer Aided Design fields. Here, the goal has been to produce an engineering model, suitable for design and analysis, which accurately reproduces the complex mechanical behavior of woven materials. Our work straddles these two categories. We are attempting to create an engineering model of cloth that draws heavily on modeling and visualization techniques that have been explored most fully by the computer graphics community.

#### 2.2.1 Computer Animation Models

The first published model of cloth for computer animation was presented by Weil [61]. He describes a two-step geometric process that models a rectangular cloth structure hanging from several constraint points. The first step of his algorithm recursively connects the constraint points with catenary curves. The second step uses a relaxation technique to enforce distance constraints between all grid points in order to create smooth cloth-like folds in the grid. Kunii and Gotoda [34] present a cloth wrinkle modeling technique that is a hybrid of physically-based and geometric approaches. They first perform a dynamic analysis on small deformable sheets and identify a limited set of characteristic deformations. These deformation primitives are then placed into a pre-defined network that describes a cloth. The wrinkle primitives then deform and respond to the changing geometric constraints acting on the network. Dhande et al. [18] also present a hybrid physically-based and geometric technique for modeling draping fabrics. They present a method for determining the parameters of a swept surface that produce a geometric model that looks like draping cloth. The values of the geometric parameters of the surface are based on the principles of applied mechanics.

The first published physically-based model of cloth was produced by Feynmann [20]. He simulated some of the mechanical properties of cloth by defining a set of energy equations, based on the theory of elastic shells, over a topologically 2-D grid of 3-D points. He minimized the energy of the grid with a stochastic
technique and a multigrid method, in order to drape his model over solid objects. Haumann and Parent [23] developed an object-oriented environment, called the Behavioral Test-Bed, with a library of simple physically-based actors. Their actors included a point mass, environmental forces, a spring connecting two point masses, a hinge that connects the two triangles formed by four point masses, and aerodynamic drag and wind actors. By performing physical simulations on simple meshes of these actors, they created several animations including a flag waving and curtains blowing in a breeze. 

Taking continuum mechanics and differential geometry as their starting point, Terzopoulos, Platt, Barr and Fleischer [53, 54, 55] developed a wide range of physically-based models for computer graphics. The goal of their work was not to accurately model specific deformable materials. Instead they were interested in creating physically-based models for computer animation that could be used to produce qualitatively familiar behaviors. With that goal in mind, they present a simplified set of equations based on elasticity theory that describe elastic and inelastic deformations, interactions with solid geometry, and fracture for flexible curves, surfaces, and solids. These equations describe the deformation energy of the models as functions of arc-length, curvature and torsion. By applying finite difference and finite element methods to these models, they created 3-D cloth-like structures that bend, fold, wrinkle, interact with solid geometry, and tear. They used their models to simulate a membrane shrinking around a jack, a flag waving, a carpet sliding over a cylinder and a sphere, and even a torn IRS 1040 form. Their model has also been implemented and extended by a team led by Magenat-Thalmann and Thalmann [14, 35, 38, 62] to simulate complete sets of clothing. Another group has used the Terzopoulos model as the foundation of a distributed force model that may be used to more accurately simulate how cloth responds to air flow [36]. Aono [2] also attempted to simulate cloth-like structures with a model based on elasticity theory. Instead of basing his model on differential geometry metrics, he takes a more conventional stress-strain analysis approach [57]. He generated several simulations by subjecting his model to impulse forces and calculating the resultant waves traveling through the cloth.

2.2.2 Engineering and CAD Models

It is surprising to note that the textile engineering community has conducted very little research in the area of modeling the large-scale structures and deformation of draping cloth. Most of its effort has focused on relating the behavior of woven materials to traditional mechanical parameters, such as Young’s modulus, bending modulus, and Poisson’s ratio. The work has focused on calculating stress-strain curves, load-extension relationships, the relationships among geometric parameters, and the dependence of bending moment on curvature. Very little work has actually utilized this mechanical information to predict the overall shape of a piece of draped fabric.

Of the few who have attempted to do this, the first work was reported in Shanahan et al. [46], who used the engineering theory of sheets, shells and plates to characterize a matrix of elastic parameters for a sheet of material. Lloyd [37] later provided non-linear extensions to the matrix and used finite element methods to simulate a 3-D circular cloth being deformed by a projectile. Eischen et al. [19] have modeled cloth structures using a large deformation beam and shell theory proposed by Simo et al. [48, 49]. By implementing the differential stress-strain equations of Simo et al. and discretizing them into nonlinear finite element equations, Eischen et al. were able to simulate the bending of cloth under a variety of situations. Given these equations and experimentally derived values for the bending rigidity, membrane rigidity and shear rigidity of woven cloth, they primarily focused on simulating cloth configurations with a symmetry that collapses the problem down to two dimensions, allowing them to model the cloth cross-section as a bending beam. They provide sketches of their 2-D simulation results and one three-dimensional simulation employing the shell theory in a plot of a stiff sheet draped over a cube.

Collier et al. [16] present a finite element approach to modeling the draping behavior of cloth. They characterize the deformations of cloth during draping as a non-linear small-strain/large-displacement problem in finite element analysis. They describe a technique for deriving and computing the non-linear small-strain/large-displacement equations from the linear small-strain/small-displacement equations by applying the variational theorems of solid mechanics. They used a 96 element and 128 node mesh to perform draping simulations, where a circular piece of cloth is draped over a cylinder. They also built their own version of the Drapemeter [15], a device that measures the draping properties of cloth, and tested a 100% cotton sample of cloth. The draping coefficients produced by their simulations compared favorably
with those measured on the actual associated cloth samples. As part of an apparel CAD system [42], Imaoka et al. [30, 31] developed a continuum mechanics model of cloth based on the large deformation shell theory of Green and Zerna [22]. They first analyzed the orthotropic stress-strain relationships of a cloth-like element by applying some of the assumptions of Shanahan et al. [46], Green’s strain formulation, and Kirchoff’s stress-resultants. These assumptions and relationships are used to derive strain energy equations for tensile, bending and shear deformations in cloth. These equations are then minimized using a form of Newton’s method to place a 3-D cloth structure in a final configuration over a solid object. They also attempted to incorporate data from the Kawabata mechanical tester into their model, but were unable to find a clear mapping of the non-linear test data into their linear model.

3 A Theoretical Particle-Based Model of Cloth

Our theoretical particle-based model approaches the problem of modeling the 3-D structures of draping cloth in a new way. Instead of looking at cloth as a continuous sheet, our model attempts to explicitly capture the micro-mechanical structural elements of cloth that are significant to its draping behavior. This model has been fully described elsewhere [9, 29], but is reviewed briefly here for completeness, since an understanding of the model is essential to the discussion of new developments reported in later sections. The description follows closely that found in [9].

3.1 Capturing Cloth Microstructure with Energy Functions

Our model is based on the idea, explained in the introduction, that cloth is not a continuous sheet, but rather is a complex mechanical structure. By representing this structure in the model, various complexities in the mechanical behavior of the material are quite naturally represented. It is neither desirable nor computationally practical to represent the full detail of the underlying thread structure of woven fabric, but we have found an intermediate position that appears to capture the most important interactions. We model cloth as a collection of particles that conceptually represents the crossing points of warp and weft threads in a plain weave, as diagramed in Figure 3.1. Several of the important mechanical interactions that determine the behavior of plain woven fabric occur at these crossing points. Most importantly, in most fabrics the compression is so great at thread crossing points that the threads are effectively clamped together, providing an axis around which bending can occur in the plane of the cloth. Other interactions that do not occur at crossing points, such as stretching within threads and out of plane bending, can be conveniently discretized and lumped at the crossing points.

In our cloth model, we represent the various constraints and interactions occurring at the thread level with energy functions that capture simple geometric relationships between particles within a local neighborhood. This is an application of the concept of enforcing geometric constraints on parameterized models with energy functions proposed by Witkin et al. [63]. Our energy functions attempt to encapsulate four basic mechanical interactions occurring at the thread level: thread contact, thread stretching, thread bending, and thread trellising. These are shown graphically in Figure 3.2, and each is represented by a term in the energy equation

$$U_{total_i} = U_{repel_i} + U_{stretch_i} + U_{bend_i} + U_{trellis_i} + U_{gravity_i}. \quad (3.1)$$

$U_{total_i}$ is the total energy of a particle $i$. $U_{repel_i}$ is an artificial energy of repulsion, that effectively keeps every other particle at a minimum distance, providing some measure of protection against self-intersection of the cloth. $U_{stretch_i}$ is the energy function that connects each particle with its four-connected neighbors, and captures energy of tensile strain. $U_{bend_i}$ is the energy due to threads bending out of the local plane of the cloth, and $U_{trellis_i}$ is the energy due to bending around a thread crossing in the plane. $U_{gravity_i}$ is the gravitational potential energy due to the distributed mass of a single particle. Contact and stretching are functions only of the interparticle distance $r_{ij}$ (Figure 3.2-I), whereas bending and trellising are functions of various angular relationships between segments joining particles (Figure 3.2-II & 3.2-III). $U_{gravity_i}$ is a function of the height ($z$ component) of the particle. The phenomenon that we call trellising requires some explanation. It occurs when threads are held fast at their crossings and bend to create an “S-curve” in the local plane of the cloth. The macroscopic phenomenon produced by this kind of deformation is related to shearing in a continuous sheet of material, but since our model treats cloth as an interwoven
grid of threads, rather than as a continuous sheet, trellising is a more descriptive term. Below, we explain how each of the five energy functions are formulated.

### 3.2 Repelling, Stretching and Gravity

At this stage of our work, we assume that the threads in the fabric do not stretch significantly when a cloth is simply draping under its own weight. Therefore, we have not attempted to use Kawabata tensile strain data, but have retained the stretching and repelling functions from our original model. There is, however, no reason why Kawabata tensile data could not be used if one wished to model fabric under tensile load. The stretching and repelling functions together provide a steep energy well that acts to keep 4-connected neighbor particles at a nominal distance $\sigma$ from each other. We have had good success with the functions given by

\[
R(r_{ij}) = \begin{cases} 
    C_0[(\sigma - r_{ij})^5/r_{ij}] & r_{ij} \leq \sigma \\
    0 & r_{ij} > \sigma,
\end{cases}
\]

and

\[
S(r_{ij}) = \begin{cases} 
    C_0[(r_{ij} - \sigma)/\sigma^5] & r_{ij} \leq \sigma \\
    0 & r_{ij} > \sigma,
\end{cases}
\]

where $C_0$ is a scale parameter that regulates the strength of repulsion $R$ and stretching $S$, and $r_{ij}$ is the distance between the current particle $i$ and a neighboring particle $j$.

The function $U_{repe}$ is calculated by summing over all particles, as given by

\[
U_{repe} = \sum_{j \neq i}^n R(r_{ij}).
\]

In practice, our simulation algorithm maintains a spatial enumeration together with the appropriate bookkeeping, so that the summation need only be done over particles that are spatially near neighbors.

An energy well is produced by directly connecting each particle with the stretching potential to its 4-connected neighbors. It is calculated by summing $S$ for each neighbor as given by

\[
U_{stretch} = \sum_{j \in N_i} S(r_{ij}),
\]
I: Repelling & Stretching

II: Bending

III: Trellising

Figure 3.2: Cloth model energy functions
where $N_i$ is the set of four-connected neighbors to particle $i$. The combined repelling and stretching functions enforce the distance constraint between neighboring particles in the grid.

The particle energy due to gravity is simply defined as

$$U_{\text{gravity}} = m_i g h_i,$$

(3.6)

where $m_i$ is the mass and $h_i$ is the height of particle $i$, and $g$ is gravitational acceleration. The mass of a particle is defined as the mass of the small patch of cloth represented by that particle.

### 3.3 Bending and Trellising

In contrast to stretching, the bending and trellising properties are significant contributors to the overall draping behavior of cloth, even when it is simply draping under its own weight. Therefore, we devoted our efforts to tuning the bending and trellising energy functions from the empirical data produced by the Kawabata System. The details of this will be presented in the next section.

We define the energy of bending as a function of the angle formed by three particles along a weft or warp “thread line”, as shown in Figure 3.2-IIa. The complete bending energy is

$$U_{\text{bend}} = \sum_{j \in M_i} B(\theta_{ij}),$$

(3.7)

where $M_i$ is the set of six angles $\theta_{ij}$ formed by the segments connecting particle $i$ and its eight nearest horizontal and vertical neighbors. This definition is used so that the derivative of bending energy with respect to the position of particle $i$ reflects the total change in bending energy due to change in position. The redundancy in this definition is taken care of later by proper scaling.

The phenomenon of trellising is mapped into the modeling cell depicted in Figure 3.2-IIIa. In the cell, two segments are formed by connecting the two pairs of neighboring particles surrounding a central particle. An equilibrium crossing angle for the segments is predefined. Currently, we assume that this is $90^\circ$, but the equilibrium angle could easily change over the course of a simulation if one were to model frictional slippage. The trellis angle $\phi$ is then defined as the angle formed as one of the line segments moves away from the equilibrium. The complete function for our energy of trellising is

$$U_{\text{trellis}} = \sum_{j \in K_i} T(\phi_{ij}),$$

(3.8)

where $K_i$ is the set of four trellising angles $\phi_{ij}$ formed around the four-connected neighbors of particle $i$. As with bending, this redundant formulation was chosen so that change in total energy with change in a single particle’s position is completely accounted for locally.

### 3.4 Initial Results

The specific energy functions that we initially used to verify the theoretical model were simply reasonable guesses. They were derived by identifying appropriate boundary conditions, and then choosing convenient functions that would interpolate these boundary conditions. We implemented our simulation of the model as a two-phase process operating over a series of small discrete time steps [11, 29]. The first phase of the simulation for a single time step models the effect of gravity, and accounts for collisions between the cloth model and a geometric model that defines the objects with which the cloth is interacting. The second phase uses a stochastic energy-minimization technique to enforce interparticle constraints and moves the configuration into a local energy minimum before continuing with the next time step. A stochastic technique is used to inject perturbations into the particle grid in order to produce a more natural asymmetric final configuration.

When the simple interactions governing the particles in the model are calculated across the whole grid of particles, they aggregate to produce a macroscopic draping behavior that is convincingly close to that of cloth. Even with our initial “sketched-in” energy functions, we were able to produce visually satisfying results such as those in Figure 3.3, which shows cloth draped over both an easy-chair and an end-table.
Our early experiments showed that our theoretical model produced convincing “cloth-like” results, confirming that we had identified and correctly implemented certain essential low-level components of cloth’s microstructure. Even though the energy functions that embodied these components were defined by an ad hoc process, and did not accurately model the physical energy of the system, we were able to fine-tune weights associated with these functions by running just a few simulations. This encouraged us about the robustness of our approach. However, even though we could generate reasonable simulation results, there were many things about our simulated cloth that we did not know. Our model was not based on physical units, so we did not know the actual size of our simulated cloth sample, and we could not query the model for any kind of mechanical information. Most importantly we could not confidently simulate particular kinds of cloth.

4 Tuning Cloth Particle Energy Functions Using Empirical Data

In order to give our model a solid physical grounding, we have recently developed a technique for deriving the energy equations of our model from the empirical mechanical data produced by the Kawabata Evaluation System [32]. This system is a standard set of equipment used to measure numerous physical and mechanical properties of cloth. By basing a key portion of our energy equations on Kawabata-derived data, our model becomes truly physical and quantitative. The incorporation of the empirical data into our theoretical model allows us to measure the mechanical properties of a particular woven material, generate the appropriate energy equations, and confidently simulate its draping behavior.

4.1 Measurements from the Kawabata Evaluation System

The Kawabata System may be used to measure the bending, shearing and tensile properties of cloth, in addition to its surface roughness and compressibility. For bending, shearing and tensile properties, the equipment measures how much force is required to perform three kinds of deformation on a fabric sample of standard size and shape. The system produces plots of each force as a function of some geometric
parameter. Typical plots for a 100% cotton sample are presented in Figures 5.1 and 5.3. The Kawabata bending plot is produced by clamping a 20 cm. × 1 cm. sample of cloth along both its long edges. The sample is then bent between the clamps and the moment necessary to accomplish the bending is recorded, as shown in Figure 4.1. By assuming that the 1 cm. cross-section bends with constant curvature, a plot of bending moment $M$ versus curvature $K$ may be produced. The shearing plot shown in Figure 5.3 was produced by applying a shearing force along one of the long edges of a 20 cm. × 5 cm. cloth sample, as schematicized in Figure 4.2. The force $F_s$ needed to shear the cloth and the resulting shear angle $\phi$ are measured. By cutting samples in two orthogonal directions out of the original cloth, it is possible to measure the shear and bending properties in both the warp and weft (filling) directions of the cloth.

We have found it is useful to think of the mechanical properties of cloth as being divided into three loosely defined regions, a region of initial resistance to deformation, a region of low deformation and a region of high deformation. The mechanical behavior of cloth throughout its range of deformation is non-linear, especially when the cloth initially begins to deform. Usually though, the mechanical properties of cloth in the low deformation region are fairly well-behaved. It is this reasonable behavior of cloth in the low deformation region that vindicates the assumption of linear elasticity made in continuum cloth models, and allows these models to produce a "cloth-like" behavior. Unfortunately, the mechanical properties in the high deformation region are not as well-behaved, and defy a simple, general mathematical description.

The data contained in the Kawabata plots only present the bending and shearing properties of cloth for small deformations. The phenomena associated with the behavior of cloth in the high deformation regions severely limits the range of measurements that can be taken with the Kawabata System. The Kawabata System attempts to measure a single deformation in a single direction. In order to do this, the measuring system makes certain assumptions about the deformations taking place while a cloth’s properties are being measured. In the low deformation region these assumptions hold and make the measurements straightforward. However, when the deformations become large, the behavior becomes more complex,
and the sample begins to buckle. Thus, the assumptions of pure bending with constant curvature, and pure planar shearing are no longer valid. The limited range of the Kawabata measurements also limits the exactness of our model. We are able to derive “exact” energy equations in those deformation ranges covered by the Kawabata System, but we must make our best educated guess for what is happening outside of that range.

4.2 Generating Energy Functions from Kawabata Data

The process of generating particle energy functions for woven cloth from Kawabata-derived data has three stages. In the first stage, the curves in the Kawabata plots are approximated with piecewise polynomial functions. In the second stage, certain physical assumptions are made about the cloth and its threads, and the measuring process, in order to derive energy functions from the piecewise approximations. Since it is only possible to measure the mechanical properties of cloth under low deformation conditions, we can only derive energy equations exactly for low shear angles and low curvature. Thus, we make assumptions about the high deformation behavior of cloth in order to generate equations valid over the entire range of deformations. In the third and final stage, the derived equations are scaled so that they will produce energy values in consistent physical units. This process is described below.

4.2.1 Approximating the Kawabata Curves

The first step in deriving energy equations from Kawabata data involves approximating the curves in the Kawabata plots with piecewise polynomial functions that interpolate the inflection points of the curve. When approximating the plots, it was decided to use the simplest polynomials possible. We therefore break up the plots into linear and quadratic segments. The curves in the Kawabata plots are sufficiently simple to allow this restriction. The boundaries between the segments are chosen at the inflection points, so that each segment defines an area of monotonic curvature. First we fit a function to an upper, more stable segment of the Kawabata curve. We then proceed to fit additional segments to the initial segment. The additional segments are defined in such a way to maintain position and slope continuity at the segment boundaries, using standard spline interpolation techniques [13]. In general, the slope of the Kawabata plots is difficult to determine at the origin. Therefore, although the first segment must pass through the origin, we do not enforce a slope constraint there.

Figures 5.1 and 5.3 present the bending and shearing properties of a particular 100% cotton cloth as measured by the Kawabata Evaluation System. The plots are produced by applying a force in one direction, releasing the force, reversing the direction of the force, and releasing the force once again. The plots clearly show the hysteretic behavior of cloth – the path of deformation when the cloth is stressed is different from the path when the stress is released, producing a loop in the plot. Hysteresis is typical in a nonconservative material, i.e., one in which energy is lost when the material is deformed. In cloth this energy loss is accounted for mostly by internal friction. Since we do not currently model the frictional interactions within cloth, our model is conservative and produces no hysteresis. The effect of this simplification is discussed later. Because our model does not exhibit hysteresis, we have only modeled the first segment of the Kawabata plots, from the origin out into the first quadrant.

A full derivation of the Kawabata curve approximation for the data in Figures 5.1 and 5.3 is provided in Appendix A.

4.2.2 Deriving the Energy Equations

The problem we are faced with after formulating the Kawabata curves as piecewise polynomials is to relate these curves to the energy equations used in our model. In the case of bending, the curves relate bending moment to curvature. In the case of trellising, the curves relate force to shear angle. What is needed is energy as a function of the bending and trellising angles available in the model. The following two sections explain how the Kawabata-derived interpolating polynomials may be used to calculate particle energies.
4.2.2.1 Bending Energy Equations

Within the low deformation region within a single thread, we assume that the theory of elastic bending beams [47] is applicable, and can be used to calculate the energy of bending. The strain energy \( dU \) due to bending stored in a segment \( dS \) of an elastic beam is given by

\[
dU = \frac{MdS}{2\rho},
\]

(4.1)

where \( M \) is the bending moment acting on the segment and \( \rho \) is its radius of curvature, which is related to curvature by the equation \( K = 1/\rho \). Within our model, each particle is separated from its 4-connected neighboring particles by the equilibrium distance \( \sigma \), which is a function of the grid dimensions and particle density. Therefore, each particle represents a \( \sigma \times \sigma \) square of cloth. This square of cloth can be thought of as a series of elastic beams (threads) lined parallel to each other. The energy of bending in one of these threads is defined by the integral

\[
U = \int_0^\sigma \frac{MK}{2} dS.
\]

(4.2)

We assume that over the length of the thread within one \( \sigma \times \sigma \) patch that the moment and the curvature are constant, simplifying the energy equation for a single thread to

\[
U = \frac{MK}{2} \sigma.
\]

(4.3)

Since \( M \) is given in units of moment per unit width of sample, we can simply multiply Equation 4.3 by the width \( \sigma \) of each patch, in order to sum up the contributions of each beam (thread) within the \( \sigma \times \sigma \) patch. The energy of bending in just one direction then becomes

\[
B = \frac{MK}{2} \sigma^2
\]

(4.4)

for each particle. This calculation is performed twice for each particle, once for bending in the warp direction, and once for bending in the weft direction.

There is still one detail in Equation 4.4 that must be addressed. Equation 4.4 is a function of curvature \( K \), and \( M \) is also a function of curvature. Given that we can only calculate angular relationships between particles, how do we determine the curvature of the cloth at one patch? Curvature at a single point can be approximated by assuming that the curvature is constant from the point to its two neighbors. Given this assumption, a circle can be fit to the three points and the circle’s curvature can be calculated [6]. The equation for the circle’s curvature may be manipulated to produce an equation that is only a function of \( \theta \), the angle formed by the three points as seen in Figure 3.2-IIa,

\[
K(\theta) = \frac{2}{\sigma} \cos(\theta/2).
\]

(4.5)

Equation 4.5 provides a satisfactory approximation except in the region where \( \theta \) is small. As the angle \( \theta \) goes to zero, \( K \) approaches the value \( 2/\sigma \). This is contrary to physical reason, since as the angle formed by the three particles goes to zero, the thread they represent bends in on itself. In order to prevent this from occurring, we modify the function for curvature in this region so that it goes to infinity as \( \theta \) goes to zero. This is done by using Equation 4.5 only for values of \( \theta \) ranging from \( 180^\circ \) to \( 45^\circ \). We then fit the function \( a/\theta + b \) to the position and slope of Equation 4.5 at \( 45^\circ \), giving the complete curvature equation

\[
K(\theta) = \begin{cases} 
-\frac{(\pi/2)^2}{2} \beta \theta + \alpha + \frac{\pi}{2} \beta, & 0 \leq \theta \leq \pi/4 \\
\frac{1}{2} \cos(\theta/2), & \pi/4 < \theta \leq \pi,
\end{cases}
\]

(4.6)

where \( \alpha = \frac{2}{\pi} \cos(\pi/8) \) is the value of Equation 4.5 at \( 45^\circ \) and \( \beta = \frac{1}{2} \sin(\pi/8) \) is the slope of Equation 4.5 at \( 45^\circ \). Having curvature go to infinity as \( \theta \) goes to zero also determines the bending behavior in the high deformation region. We currently extrapolate the final segment of the bending moment vs. curvature curve into the high deformation region. Having curvature go to infinity, then prevents the threads from bending in on themselves.

To summarize, in order to calculate the energy of bending in one direction around a particle, first the angle formed by the particle and its two neighbors along a thread line is calculated. The curvature is
calculated with Equation 4.6, and is inserted into the bending moment vs. curvature functions that are derived from the Kawabata plots. Finally, the resulting bending moment and curvature are placed into Equation 4.4 to produce the energy of bending in one direction. The same steps are taken for the energy of bending in the orthogonal direction in order to produce the total energy of bending for one particle.

4.2.2.2 Trellising Energy Equations

Producing the trellising energy equations is a bit simpler than the bending energy equations. The work \( W \) produced by a force \( F \) acting over a displacement \( dS \) is defined by

\[
W = \int Fds.
\]

(4.7)

Given this simple equation we can calculate the energy stored in the 20 cm. \( \times \) 5 cm. cloth sample that is sheared in a Kawabata Shear Tester. If we assume that the width \( l \) of the sample remains constant during shearing, then the path traveled by the point at which the shearing force is applied is a circular arc whose length is defined by \( S = l\phi \), where \( \phi \) is the shearing angle. From this relationship it is easily seen that \( dS = ld\phi \). If the force point is moving along a circular arc, its direction is not always parallel to the applied force. \( F \cos(\phi) \) gives the component of the applied shearing force that is in the direction of the motion of the force point. Using this result in Equation 4.7 produces an equation for shearing energy as a function of shear angle that was first derived by Cusick [17],

\[
T = \int F \cos(\phi)ld\phi.
\]

(4.8)

Now for a specific material, we can use our Kawabata-based curves for shearing force \( F \) as a function of shearing angle \( \phi \), and integrate to produce a shearing energy equation.

Once again we are faced with the problem of defining energy curves in the high deformation region not covered by the Kawabata data. Skelton [51] states that most woven materials cannot shear more than 45°. We enforce this constraint by making the trellising energy curve go to infinity at about 60°. The extra 15° allows enough of a buffer around the singularity to ensure that the material can shear all the way to 45° if necessary. The constraint is implemented by fitting the function \( a/(1.05 - \phi) + b \) to the slope and position at the endpoint of the Kawabata-derived energy curve (the magic number 1.05, is simply a rough approximation to 60° = \( \pi/3 \) radians).

4.2.3 Adjusting the Units

Up to this point close attention has not been paid to the physical units associated with our energy equations. In order to have consistent energy units across all of the equations, we convert our equations into CGS (centimeter, gram, second) units. From the Kawabata bending plot (Figure 5.1) and Equation 4.4 we see that the units for bending energy are \((gf/cm)(1/cm)(cm)(cm)\), which gives \( gf \cdot cm \). Gram-force (gf) is the gravitational force that one gram of mass experiences at sea level. That force in dynes (the CGS unit of force) is \( 1 \ g \cdot 978.80 \ cm/s^2 = 978.80 \ dynes \). Therefore \( 1 \ gf \cdot cm = 978.80 \ dyne \cdot cm = 978.80 \ ergs \). Consequently, scaling the bending energy function, Equation 4.4, by 978.80 will produce an energy in ergs.

In the Kawabata shearing plot (Figure 5.3) the shearing force is given in units of \( gf/cm \). As with the bending energy, multiplying the force by 978.80 will convert \( gf/cm \) into dyne/cm. The \( 1/cm \) units come from the fact that the shearing force applied to the sample is divided by the length of the sample (20 cm.). Multiplying the force by an additional 20 cm. gives an absolute force in dynes. Since the trellising energy is produced by multiplying force times distance in centimeters, the energy units is dyne-cm or ergs. Recall that one particle represents a \( \sigma \times \sigma \) patch of cloth, so we would like the trellising energy per unit area. This can be computed by dividing the total energy stored in the measured sample by the area of the sample (100 cm²). Therefore the final scale factor that converts the trellising energy into \( ergs/cm^2 \) is

\[
1 \ (gf/cm) \cdot cm \cdot 978.80 \ dynes/gf \cdot 20 \ cm/100 \ cm^2 = 195.76 \ ergs/cm^2.
\]

(4.9)

In order to calculate the trellising energy of a particle, the trellising angle \( \phi \) defined by its four neighbors is calculated. The value of \( \phi \) is used in the appropriate energy equation, and the result of that evaluation

13
is then multiplied by 195.76\sigma^2. This gives the shearing energy in ergs for the \( \sigma \times \sigma \) patch of cloth represented by the particle.

5 Experimental Results

All together we have derived energy equations for three samples of material, the 100\% cotton to which we have already referred, a 100\% wool, and a polyester/cotton blend. The Kawabata plots associated with the wool and cotton/polyester samples may be found in Appendix C, and the particle energy equations derived from these plots are contained in Appendix B.

Given the energy equations for all three samples, we performed two sets of experiments to verify that they had captured the characteristic mechanical and draping behavior of each type of cloth. The first experiment recreated the Kawabata Bending and Shear Testers in simulation. From these simulations, we generated simulated Kawabata plots that demonstrate the mechanical properties of our cloth model. The second experiment was to drape real cloth over a cube and then perform the same drape in simulation, using computer visualizations of the simulation results to make visual comparisons with the actual cloth.

5.1 Simulating the Kawabata Testers

The general approach to recreating the Kawabata testers in simulation and subsequently generating simulated Kawabata plots involves first incrementally deforming the cloth model as a real Kawabata tester would deform a real cloth sample. At each increment of the deformation, the total energy of the simulated cloth sample is calculated using the Kawabata-derived energy functions. Using these incremental energy values, an incremented geometric parameter, and the same physical assumptions used to generate the energy equations, the mechanical property measured on real Kawabata equipment may be calculated from the simulation.

5.1.1 The Kawabata Bending Tester

The Kawabata Bending Tester is simulated by taking a 20 cm. \( \times \) 1 cm. simulated cloth sheet and incrementally changing the angle between particles along the thread lines in the 1 cm. direction, so that the sample bends with a constant curvature. By setting each particle bending angle \( \theta \) to

\[
\theta = 2 \arccos(\sigma K/2),
\]

(the inverse of Equation 4.5) the cloth sample may be positioned with a constant curvature \( K \) in that particular thread direction. Given Equation 5.1, the simulation is performed by incrementing the value of curvature \( K \), calculating the appropriate angle \( \theta \), setting the position of each particle so that they form an angle \( \theta \) in the direction of bending, and then calculating the total energy of the cloth. This procedure is followed for equal increments of curvature from 0 to 2.5. Given energy as a function of curvature, bending moment \( M \) can be calculated with

\[
M = \left[1/(20 \cdot 978.80)\right] \frac{2B}{SK},
\]

where \( S \) is equal to 1 cm., and the factor \(1/(20 \cdot 978.80)\) converts the units from dyne-cm to gf-cm/cm. The output of the simulated Kawabata Bending Tester for our cloth model of 100\% cotton is presented in Figure 5.2. The plot reliably approximates the first segment of the bending hysteresis loop presented in Figure 5.1.

5.1.2 The Kawabata Shear Tester

The Kawabata Shear Tester may be simulated by taking a 20 cm. \( \times \) 5 cm. simulated cloth sheet and incrementally changing the trellising angle for each particle in the sample. At each increment of the trellising angle from 0\(^\circ\) to 8.1\(^\circ\), the derivative of the trellising energy \( dI/d\phi \) is approximated by calculating the
Figure 5.1: Kawabata Bending Plot (100% cotton)

Figure 5.2: Plot from a simulated Kawabata Bending Tester (100% cotton)
Figure 5.3: Kawabata Shear Plot (100% cotton)

Figure 5.4: Plot from a simulated Kawabata Shear Tester (100% cotton)
trellising energy at two points around the current trellising angle and subsequently calculating $\Delta T/\Delta \phi$. This provides a trellising energy derivative as a function of shearing (trellising) angle. The shearing force that is required to produce the shearing deformation may then be calculated with

$$F_s = \frac{[1/(20 \cdot 978.80)] \Delta T}{\Delta \phi} / (l \cos(\phi)),$$

(5.3)

where $l$ is 5 cm. and the factor $1/(20 \cdot 978.80)$ converts dynes into gf/cm. The output of our simulated Kawabata Shear Tester for our cloth model of 100% cotton is presented in Figure 5.4, and reliably approximates the first segment of the shear hysteresis loop presented in Figure 5.3. These results clearly demonstrate that our cloth model is able to accurately reproduce the mechanical properties of cloth measured by the Kawabata System in the low deformation regions.

5.2 Draping Experiments

To further confirm the validity of our model, we conducted a series of draping experiments and associated draping simulations. The draping simulations utilized the previously derived energy equations, based on the Kawabata plots provided by the Grundy Center for Textile Evaluation at the Philadelphia College of Textiles and Science. In the actual draping experiments, 1 m. x 1 m. sections of our three cloth samples were draped over a .5 m. x .5 m. x .5 m. cube. The results of these drappings were photographed and are presented in the adjacent figure. The different views of the samples were chosen to highlight the unique draping structures of each cloth.

The same scenario was recreated in simulation. Models of 1 m. x 1 m. samples of 100% cotton, 100% wool and a polyester/cotton blend, represented by a 51 x 51 particle grid, were draped over a .5 m. x .5 m. x .5 m. cube. An entire simulation, starting with a flat cloth placed above the cube and falling to its final draped configuration, required 1 CPU-week on an IBM RS/6000 workstation. Visualizations of those draping simulations are presented in the adjacent figure. Even though the results of the actual and simulated drappings are not exactly the same, it can be clearly seen that each kind of material has a characteristic draping behavior that is captured by the simulation. The 100% cotton cloth develops a single large structure that comes out from the corner of the cube at a 45° angle. The bending stiffness of the wool sample is significantly less than the cotton’s. Therefore, it does not have the bending strength needed to support a single large fold at the corner, and the corner structure collapses into two smaller folds. The polyester/cotton blend produces the most interesting draping structures. The bending stiffness of the blend is significantly greater in the warp than in the weft direction. The effect on the draping of the cloth can be clearly seen. Since bending is so much stronger in one direction than another, the draping structure is literally pushed around the corner by the warp threads. This produces an asymmetric structure that wraps around the corner of the cube.

6 Discussion

Both the simulated Kawabata tests and the draping experiments support the validity of our particle-based cloth model. The simulated Kawabata plots demonstrate that the model is capable of generating detailed mechanical data that accurately describe real materials. The simulated plots may not look exactly like the real plots. They are only as good as the underlying approximation to the original data. If the piecewise polynomial approximations of the actual Kawabata plots do not capture every subtle bump in the real data, it cannot be reproduced in simulation. For example, in the 100% wool and the polyester/cotton blend we assumed that the shearing properties were isotropic. We did not represent the subtle differences between the warp and weft shearing curves in these materials, because we did not believe that they were important. There is no reason that those details could not be represented in the model. Overall, the important aspect of the simulated Kawabata tests is that they demonstrate that the model easily captures the non-linear mechanical behavior of woven cloth as measured by the Kawabata Evaluation System.

The draping experiments show that the model can not only produce accurate detailed mechanical data under structured conditions, but can also be used to reproduce the general draping behavior of specific
Figure 6: Actual (left) vs. simulated (right) cloth drape
materials in a more general environment. Certainly the cloth simulations do not exactly produce the actual draping configurations. This, in a sense, would be impossible, considering the chaotic and stochastic nature of draping a cloth over an object. Since a tablecloth will never fall in exactly the same way every time it is draped, it is unrealistic to expect that a stochastic simulation would be able to precisely reproduce any one draping configuration. What was found when working with real materials is that specific materials do have a general draping behavior. The stiffness of the cotton sample usually ensured that it would produce a single draping structure at its corners. The polyester/cotton blend always produced some kind of asymmetric fold that would wrap around its corners. The wool could form either a single fold or could collapse into more than one fold. Of course, each one of these materials can be forced into many kinds of draping configurations, but when allowed to drape naturally, they usually produce characteristic draping structures.

One of the advantages of using a structural model is that it provides one with the opportunity to explore and analyze the low-level structural attributes of the simulation solutions produced by the model. This low-level information provided us with insight into the free draping behavior of our samples. For example, in their final equilibrium state all of the mechanical parameter values of all the particles lie within the parameter range covered by the Kawabata-derived data. It is encouraging to note that the draping behavior of woven cloth is based on the low deformation mechanical properties that may be attained from the Kawabata System. The estimated Kawabata curves in the high deformation region are defined for completeness and are possibly needed during the transient stage of the simulation. They are apparently unimportant for the steady-state final configuration.

The low-level model data also allowed us to explore the differences in the draping behavior between the 100% cotton and the polyester/cotton samples. The Kawabata plots of each sample strongly exhibit anisotropic bending properties, but their draping behaviors do not exhibit the same anisotropy. The cotton sample forms symmetric draping structures at the corners of the cube, implying a balance of warp and weft bending forces. The polyester/cotton sample clearly exhibits anisotropic bending by producing asymmetric draping folds that imply that the cloth is stiffer in the warp direction. Examining the geometric particle parameters shows that the cotton is stiffer than the polyester/cotton and therefore has lower curvature values. This implies that the stiffness of the cotton prevents the cloth from bending into those curvature regions where the anisotropy is significant. The polyester/cotton is less stiff and bends more into the curvature regions where the differences between the warp and weft bending are more pronounced. Another important attribute to examine is the materials’ initial resistance to bending, which can be estimated by the slope of the Kawabata bending curves at the origin. The slope of the cotton bending curves at the origin is quite high, implying a significant initial resistance to bending. The polyester/cotton sample has a lower slope and therefore a lower initial resistance to bending. This lower resistance allows the cotton/polyester particles to bend into the curvature regions where the effects of the anisotropic bending properties become more evident. Conversely, the cotton sample’s stiffness limits its bending; thus preventing its high curvature anisotropy from clearly appearing.

The differences evident between the simulated and real drappings should also be addressed. The main difference between the two appears to be the sharpness of the edges and the corners. The simulated samples appear to have “soft” edges as if being draped over a rounded cube, when in fact they were not. The top of the cotton draping structure in the actual cotton sample comes to a fine point, while in the simulation, it is a bit wider. Both of these differences are related to the fineness of the particle grid. A $51 \times 51$ particle grid is capable of producing large-scale draping structures, but it is not sufficient for capturing the sharp bends over the edges or at the corners of a cube. We believe that utilizing a finer particle grid along with an adaptive sampling algorithm will remove these differences. At this time, it is not computationally feasible to increase the particle resolution beyond $51 \times 51$.

The issue of computational speed is one that we chose to ignore for a period while we concentrated on developing and proving the methodology. We do not feel that this is a fundamental problem with our approach, but rather one that simply needs some attention. We see four ways to increase the computational speed of our approach: 1) improving the computational environment, 2) improving the energy minimization technique, 3) approximately predraping the material, and 4) exploiting massive parallelism. The first, and most obvious way to speed up our simulations would be to write custom simulation code. Currently we work in an object-oriented, message-passing environment that has been excellent for rapid prototyping, but entails a heavy overhead [8, 21]. A more fundamental speed improvement can be had by improving our simulation techniques. The slowest part of our simulation is the energy mini-
mization calculation at each time step. We currently use a stochastic technique that roughly follows a numerically-determined approximation to the energy gradient at each particle. We have recently begun work custom-coding an efficient implementation of our model, that uses precalculated tables to more exactly and efficiently determine energy gradients, and are experimenting with a pure gradient-descent approach to energy minimization. Preliminary results show speed-ups of at least an order of magnitude. The idea of using an approximate, purely-geometric predrape, followed by use of a physical model to perfect the drape has already been tried with success [42, 62]. This would significantly reduce the simulation time needed to get the fabric into position over a geometric model, leaving only the calculations needed for a final energy minimization step. Massive parallelism has been shown to be an especially efficient way of computing uncoupled particle systems [50]. The highly distributed form of our particle model, with its simple local computations and well defined neighbor interactions, should also be especially amenable to this computational approach [28].

To designers, one of cloth’s most important characteristics is its ability to be shaped and creased. Our model, as it stands, is conservative – no energy is lost during a deformation. The consequence of this is that we cannot yet mimic shaping and creasing. This has not been an important issue in the kinds of free-draping studies that we have conducted, but would be of very great importance when looking at fabric under the high stresses that occur in manufacturing. It should, however, be possible to extend our model in a natural way to simulate non-conservative deformation. The stretching, bending, and trellising energy functions all either explicitly or implicitly represent a “rest” value for their independent variables. It would be straightforward to represent all of these rest values explicitly, and then vary them as a function of local strain during the course of a simulation. This would mimic the local effects of frictional slippage within the weave. This could be put on a firm physical basis, at least for the low deformation region, by adjusting “slippage” so that the full hysteresis curves from the Kawabata tester are matched.

7 Conclusion

We have presented a particle-based model capable of predicting the static draping behavior of specific kinds of woven cloth. The model treats cloth as a complex mechanism, rather than a continuous sheet, and can be tuned to simulate different types of fabrics. Numerous experiments have been conducted in order to verify our approach. There are several significant aspects to the work presented here. It has demonstrated that a microstructural model may be used to reproduce the macroscopic mechanical behavior of real flexible materials. We have shown that the use of such an approach allows for the straightforward incorporation of non-linear empirical data into the model. The model has been verified by two experiments. One generates the low-level mechanical properties of real fabrics. The other recreates the distinctive macroscopic geometric structures of draping cloth and compares them to actual cloth drappings. This kind of evidence has not been presented in previous cloth modeling studies. Our model should be useful in the computer-aided design of garments and other woven structures. With our new approach real materials may now be measured, the empirical data used to derive energy equations, and the draping behavior of specific materials may then be confidently simulated on a computer.

8 Acknowledgements

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A Deriving the Particle Energy Equations for 100% Cotton

The warp Kawabata bending curve for the cotton sample found in Figure 5.1 has been split into three segments. The lower two segments are approximated with quadratic functions. An upper segment beyond the empirical curve is defined as linear. The endpoints for each segment are placed at an inflection point (i.e., curvature equal zero) in the curves. The segments are defined so as to maintain position and slope continuity at the segment boundaries. A quadratic function has been fit through the following three points on the cotton sample’s warp bending curve,

\[(.4, .23), \ (1.2, .45), \ (2.52, .76).\]

A quadratic curve is fit to the endpoint, \((.4, .23)\), and slope, \(.29015151\), of this segment. The other end of the lower curve goes through the origin with no slope constraint. A linear function is then fit to the upper endpoint, \((2.52, .76)\), and slope, \(.20984848\), of the initial quadratic segment. The complete piecewise polynomial approximation of the Kawabata warp bending plot for the cotton sample is

\[(warp) : M = \begin{cases} 
-7.1212123K^2 + .85984849K & 0 \leq K \leq .4 \\
-0.18939394K^2 + .30330383K + .11090909 & .4 < K \leq 2.52 \\
.20984848K + .231118183 & K > 2.52.
\end{cases} \quad (A.1)\]

The upper linear segment is necessary, because the middle segment is in the form \(-aK^2 + bK + c\). The negative coefficient on the \(K^2\) term may cause trouble, because at some high value of \(K\), M will become negative producing unnatural results.

The weft cotton bending curve is split into two segments. The upper segment is linear and the lower segment is quadratic. The linear function is fit through the points

\[(.5, .14), \ (2.52, .4).\]

A quadratic curve is fit to the endpoint, \((.5, .14)\), and slope, \(.12871287\), of this segment. The other end of the lower curve goes through the origin with no slope constraint. The complete piecewise polynomial approximation of the Kawabata weft bending plot for the cotton sample is

\[(weft) : M = \begin{cases} 
-30257426K^2 + .43128713K & 0 \leq K \leq .5 \\
.12871287K + .075643564 & K > .5.
\end{cases} \quad (A.2)\]

The Kawabata shearing plot for the cotton sample may be found in Figure 5.3. The warp shearing curve is split into two segments. The upper segment is linear and the lower segment is quadratic. The linear function is fit through the points

\[(.5, 2.3), \ (7.5, 15.2).\]

The shearing angles must be converted to radians. The points then become

\[(.00873, 2.3), \ (.13, 15.2).\]

A quadratic curve is fit to the endpoint, \((.00873, 2.3)\), and slope, \(106.37421\), of the linear segment. The other end of the lower curve goes through the origin with no slope constraint. The complete piecewise polynomial approximation of the Kawabata warp shearing plot for the cotton sample is

\[(warp) : F_s = \begin{cases} 
-17993.714\phi^2 + 420.544446\phi & 0 \leq \phi \leq .00873 \\
106.37421\phi + 1.3713532 & .00873 < \phi < .13.
\end{cases} \quad (A.3)\]

The weft cotton shearing curve is split into two segments. The upper segment is linear and the lower segment is quadratic. The linear function is fit through the points
(5, 2.3), (8.2, 17.6).

By converting the shear angles into radians, the points then become

(0.0873, 2.3), (0.143, 17.6).

A quadratic curve is fit to the endpoint, (0.0873, 2.3), and slope, 113.94950, of the linear segment. The other end of the lower curve goes through the origin with no slope constraint. The complete piecewise polynomial approximation of the Kawabata weft shearing plot for the cotton sample is

\[
(wef t) : F_\phi = \begin{cases} 
-17125.983\phi^2 + 412.96917\phi & 0 \leq \phi \leq 0.0873 \\
113.94950\phi + 1.3052208 & 0.0873 < \phi \leq 0.143.
\end{cases} \quad (A.4)
\]

In order to produce trellising energy \( T \), Equation 4.8, \( \int F_\phi \cos(\phi) \phi d\phi \), must be evaluated. The previous two shearing force equations are integrated and the function \( a/(1.05 - \phi) + b \) is fit to the slope and position at the endpoint of the upper segment of the resulting function. This final segment prevents the trellising angle from becoming greater than 60° (~1.05 radians). Within the lower segment \( \sin(\phi) \) approximately equals \( \phi \) and \( \cos(\phi) \) approximately equals 1. By utilizing these approximations, the lower segment of \( T \) takes the form \( a\phi^2 + b\phi^2 \). The complete warp trellising energy function is

\[
(warp) : T = \begin{cases} 
-29991.982\phi^3 + 1051.3683\phi^2 & 0 \leq \phi \leq \phi < 0.0873 \\
531.87105(\cos(\phi) + \phi \sin(\phi)) + 6.856766\sin(\phi) & 0.0873 < \phi \leq 0.13 \\
63.783609/(1.05 - \phi) - 63.985748 & \phi > 0.13.
\end{cases} \quad (A.5)
\]

The weft trellising energy function is

\[
(wef t) : T = \begin{cases} 
-28549.108\phi^3 + 1032.4738\phi^2 & 0 \leq \phi \leq 0.0873 \\
569.7475(\cos(\phi) + \phi \sin(\phi)) + 6.5261010\sin(\phi) & 0.0873 < \phi \leq 0.143 \\
71.654187/(1.05 - \phi) - 72.294606 & \phi > 0.143.
\end{cases} \quad (A.6)
\]

B Additional Energy Equations

B.1 100% Wool

The functions that approximate the bending moment curves for the 100% wool in Figure C.1 are:

\[
(warp) : M = .05K. \quad (B.1)
\]

\[
(wef t) : M = .058K. \quad (B.2)
\]

The function that approximates the shearing force curve for 100% wool in Figure C.2 is:

\[
F_\phi = \begin{cases} 
-57344.509\phi^2 + 214.63860\phi & 0 \leq \phi \leq .00175 \\
55.753231\phi^2 + 13.737688\phi + .1757830 & .00175 < \phi \leq .147.
\end{cases} \quad (B.3)
\]

The trellising energy function derived from the shearing force function is:

\[
T = \begin{cases} 
-95160.862\phi^3 + 535.51110\phi^2 & 0 \leq \phi \leq .00175 \\
278.76616\phi^2 \sin(\phi) - 556.65337\sin(\phi) & 0.00175 < \phi \leq .147 \\
+557.53231\phi \cos(\phi) & \phi > .147.
\end{cases} \quad (B.4)
\]
B.2 Polyester/Cotton Blend

The functions that approximate the bending moment curves for the polyester/cotton blend in Figure C.3 are:

\[
(warp) : M = \begin{cases} 
-0.10372093K^2 + 0.16186047K & 0 \leq K \leq 0.5 \\
0.05813935K + 0.02591033 & K > 0.5.
\end{cases}
\]  

(B.5)

\[
(weft) : M = \begin{cases} 
-0.04883721K^2 + 0.07418605K & 0 \leq K \leq 0.5 \\
0.025581395K + 0.012209302 & K > 0.5.
\end{cases}
\]  

(B.6)

The function that approximates the shearing force curve for the polyester/cotton blend in Figure C.4 is:

\[
F_s = \begin{cases} 
-5008.0931\phi^2 + 123.390393\phi & 0 \leq \phi \leq 0.00873 \\
100.07324\phi^2 + 34.715344\phi + 0.38930818 & 0.00873 < \phi \leq 0.143.
\end{cases}
\]  

(B.7)

The trellising energy function derived from the shearing force function is:

\[
T = \begin{cases} 
-8347.5746\phi^3 + 309.76204\phi^2 & 0 \leq \phi \leq 0.00873 \\
500.3662\phi^2\sin(\phi) - 998.78586\sin(\phi) + 1000.7324\phi \cos(\phi) & 0.00873 < \phi \leq 0.143 \\
+ 173.57672(\cos(\phi) + \phi \sin(\phi)) - 173.58239 & .00873 < \phi \leq .143 \\
30.127330/(1.05 - \phi) - 30.694314 & \phi > .143.
\end{cases}
\]  

(B.8)
C Additional Kawabata Plots
Figure C.1: Kawabata Bending Plot (100% wool)

Figure C.2: Kawabata Shear Plot (100% wool)
Figure C.3: Kawabata Bending Plot (polyester/cotton)

Figure C.4: Kawabata Shear Plot (polyester/cotton)