Calculating Center of Mass in an Unbounded 2D Environment

Linge Bai and David Breen
Department of Computer Science, College of Engineering

Introduction

- We study the behavior of simple, 2D, self-organizing primitives that interact and move in an unbounded environment to create aggregated shapes.
- Each primitive is represented by a disk and a unit point mass. In order to compare the aggregated shape produced by the primitives to other shapes, the centers of mass of the two shapes must be aligned.
- We present an algorithm for calculating the center of mass (COM) for a set of point masses that are distributed in an unbounded 2D environment.
- The algorithm calculates the centroid for each coordinate component separately by forming two "orthogonal" tubes, calculating a center-of-mass in 3D for each tube and then projecting the 3D COM back onto the tubes, in order to produce the 2D COM of the points.

Motivation

- In general the center of mass COM of a set of point masses is calculated by the weighted average of all the points as in the equation

$$COM = \frac{\sum m_i X_i}{\sum m_i}, \qquad (1)$$

where m_i is the mass of point i and X_i is its location.
- The point masses exist in an unbounded environment and the problem is complicated by the lack of a proper origin for the environment's coordinate system.
- The general method does not produce the correct result in a 2D unbounded environment.

Results from the general method

- Equation 1 cannot be used to provide a solution, since the collection of points may be aggregated across the environment's boundaries.
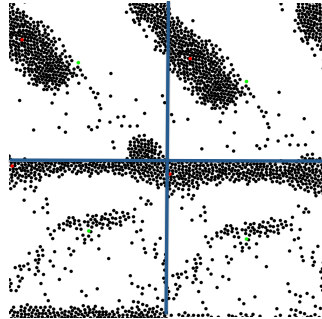

- Green dot: COM computed with the general method; red dot: COM computed with our method.

Algorithm Overview

- The algorithm calculates the centroid for each coordinate component separately by forming two tubes, and calculating a center-of-mass in 3D for the points on each tube.
- The 3D COM is then projected back onto the tubes. The location of the projected COM in the coordinate system of the tube surface is the 2D COM of the points.

A 1D Demonstration
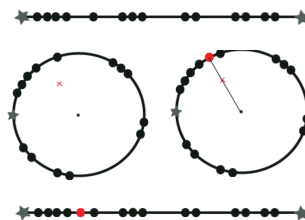
Algorithm Description

- The location of a point mass in the 2D rectangle is represented by the 2D Cartesian coordinate (i, j), with values ranging from (0, 0) to (i_{max}, j_{max}).
- The first tube (T_i) is created by connecting the i = 0 edge of the rectangle with the i = i_{max} edge. The second tube (T_j) is created by connecting the j = 0 edge of the rectangle with the j = j_{max} edge.
- The 2D to 3D transformation for points on tube T_i is defined by

$$x = r_i \cos(\theta_i), \qquad y = j, \qquad z = r_i \sin(\theta_i),$$
$$r_i = \frac{i_{max}}{2\pi}, \qquad \theta_i = \frac{i}{i_{max}} 2\pi. \qquad (2)$$

- The 2D to 3D transformation for points on tube T_j is defined by

$$x = i, \qquad y = r_j \cos(\theta_j), \qquad z = r_j \sin(\theta_j),$$
$$r_j = \frac{j_{max}}{2\pi}, \qquad \theta_j = \frac{j}{j_{max}} 2\pi. \qquad (3)$$

- 3D center of mass of these transformed points is calculated,

$$\bar{\mathbf{X}} = \frac{1}{n} \sum_{k=1}^{n} \mathbf{X}_k. \qquad (4)$$

- $\bar{\mathbf{X}}$ is then projected back onto each tube. We calculate just one of the COM coordinate from each tube.
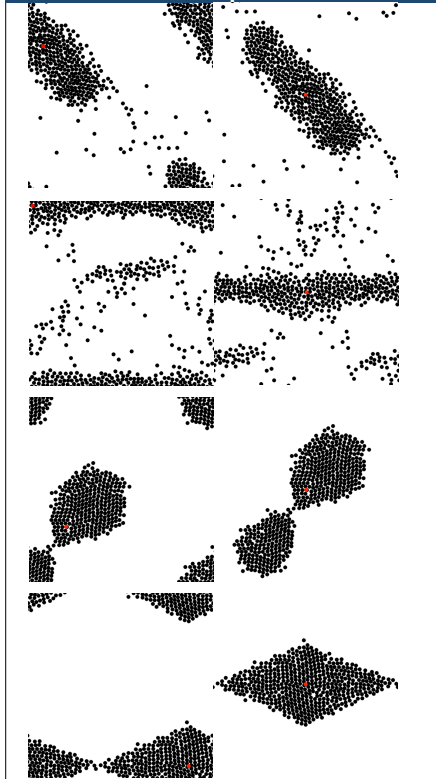- $\bar{i}$ is calculated from $\bar{\mathbf{X}}$ for Tube T_i.

$$\theta_i = atan2(-\bar{z}, -\bar{x}) + \pi, \qquad \bar{i} = \frac{i_{max}}{2\pi} \theta_i. \quad (5)$$

- $\bar{j}$ is calculated from $\bar{\mathbf{X}}$ for Tube T_j.

$$\theta_j = atan2(-\bar{z}, -\bar{y}) + \pi, \qquad \bar{j} = \frac{j_{max}}{2\pi} \theta_j. \quad (6)$$

Examples

Related Work

- One existing solution, based on SLERP, computes a weighted spherical average on the surface of a torus.
- The COM of the point masses is the point that minimizes the sum of weighted distance between COM and each point mass.

Related Publications

Linge Bai and David Breen, "Calculating Center of Mass in an Unbounded 2D Environment," Journal of Graphics Tools, Vol. 13. No. 4, pp. 53-60, 2008.