1. **Solid Model**

- A solid model unambiguously divides 3D space into a finite inside region and an infinite outside region.

2. **Overview**

- 3D solid model representations
  - Implicit models
  - Superquadrics
  - Blobbies
  - Swept objects
  - Boundary representations
  - Spatial enumerations
  - Distance fields
  - Quadtree/octrees
  - Stochastic models

3. **Implicit Solid Modeling**

- Idea:
  - Represents solid as the set of points where an implicit global function takes on certain values
  
  - Usually:
    - $F(x,y,z) < 0$, points inside of object
    - $F(x,y,z) = 0$, points on object's surface
    - $F(x,y,z) > 0$, points outside of object

- Primitive solids are combined using CSG
- Composition operations are implemented by functions which provide an implicit function for the resulting solid

From M. Ganter, D. Storti, G. Turkiyyah @ UW

4. **Quadratic Surfaces**

- Sphere
  \[ x^2 + y^2 + z^2 = r^2 \]
- Ellipsoid
  \[ \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \]
- Torus
  \[ \sqrt{x^2 + y^2} = a \]
- General form
  \[ a^2 - x^2 + b^2 - y^2 + c^2 - z^2 = 2f(x,y,z) \]

5. **Superellipsoid Surfaces**

- Generalization of ellipsoid
- Shape parameters $a$, $b$, and $c$
  \[ \left( \frac{x}{a} \right)^{2/3} + \left( \frac{y}{b} \right)^{2/3} + \left( \frac{z}{c} \right)^{2/3} = 1 \]

- Take absolute value of $x$, $y$, and $z$ to avoid exponentiating negative numbers
- If $a = b = c$ then regular ellipsoid
- Has an implicit and a parametric form

From Wei-Hwa Hung et al. @ UW
Superellipsoid Surfaces

The general superellipsoid has a parametric representation in terms of surface parameters $n, m, u, v, w, x, y, z$:

$$
\begin{align*}
    x(u,v) &= A_1(v,s_1)u(s_2) \\
    y(u,v) &= B_1(v,s_1)u(s_2) \\
    z(u,v) &= C_1(v,s_1)u(s_2)
\end{align*}
$$

where the auxiliary functions are

$$
\begin{align*}
    A_1(u,m) &= \text{sgn}(\sin u)/\sin u^{m} \\
    B_1(u,m) &= \text{sgn}(\sin u)/\sin u^{m} \\
    C_1(u,m) &= \text{sgn}(\sin u)/\sin u^{m}
\end{align*}
$$

and the sign function $\text{sgn}(x)$ is

$$
\begin{align*}
    \text{sgn}(x) &= \begin{cases} 
        -1, & x < 0 \\
        0, & x = 0 \\
        1, & x > 0
    \end{cases}
\end{align*}
$$

CSG with Superquadrics

Blobby Objects

- Do not maintain shape, topology
  - Water drops
  - Molecules
  - Force fields
- But can maintain other properties, like volume

CSG with Superellipsoids

Gaussian Bumps

- Model object as a sum of Gaussian bumps/blobs

$$
\begin{align*}
    f(x,y,z) = \sum_b \delta e^{-\frac{r^2}{2T}} - T = 0
\end{align*}
$$

- Where $r^2 = x^2 + y^2 + z^2$ and $T$ is a threshold.

Metaballs (Blinn Blobbies)
Ray-traced Metaballs

Implicit Modeling System
U. of Calgary

- Combine "primitives"
  - Points, lines, planes, polygons, cylinders, ellipsoids
- Calculate field around primitives
- View iso-surface of implicit function

Implicit Modeling System
U. of Calgary

Sweep Representations

- An alternative way to represent a 3D object
- Idea
  - Given a primitive (e.g., polygon, sphere)
  - And a sweep (e.g., vector, curve...)
  - Define solid as space swept out by primitive

Sweep Representations

- Issues:
  - How to generate resulting surface?
  - What about self-intersections?
  - How to define intersection?

Approximate Representations

- Idea: discretize the world!
- Surface Models
  - Mesh, facet and polygon representations
- Volume Models
  - Spatial enumeration
  - Voxelization
Examples

• From exact to facets....

Boundary Representation
Solid Modeling

• The de facto standard for CAD since ~1987
  - BRep integrated into CAGD surfaces + analytic surfaces +
    boolean modeling
• Models are defined by their boundaries
• Topological and geometric integrity constraints are
  enforced for the boundaries
  - Faces meet at shared edges, vertices are shared, etc.

Let’s Start Simple:
Polyhedral Solid Modeling

• Definition
  – Solid bounded by polygons whose
    edges are each a
    member of an even
    number of polygons
  – A 2-manifold: edges
    members of 2
    polygons

Properties of 2-Manifolds

• For any point on the boundary, its
  neighborhood is a topological 2D disk
• If not a 2-manifold, neighborhood not a disk

Euler’s Formula

• For simple polyhedron (no holes):
  #Vertices - #Edges + #Faces = 2
• If formula is true the surface is closed

Euler’s Formula (Generalized)

#Vertices - #Edges + #Faces - #Holes_in_faces - #Components = Genus
• Genus is the # holes through the object
• Euler Operators have been the basis of several
  modeling systems (Mantyla et al.)
Euler Operators

<table>
<thead>
<tr>
<th>Operator Name</th>
<th>Meaning</th>
<th>V</th>
<th>E</th>
<th>F</th>
<th>L</th>
<th>S</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>MEV</td>
<td>Make an edge and a vertex</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MFE</td>
<td>Make a face and an edge</td>
<td>1</td>
<td>1</td>
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</tr>
<tr>
<td>MSGF</td>
<td>Make a shell, a face and a vertex</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MSG</td>
<td>Make a shell and a hole</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MEL</td>
<td>Make an edge and kill a loop</td>
<td>0</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Loop L → H, Shell S → C

Steps to Creating a Polyhedral Solid Modeler

• Representation
  – Points, Lines/Edges, Polygons
• Modeling
  – Generalization of 3D clipping to non-convex polyhedra, enables implementation of booleans

State of the Art: BRep Solid Modeling

• ... but much more than polyhedra
• Two main (commercial) alternatives
  – All NURBS, all the time
    • Pro/E, SDRC, ...
  – Analytic surfaces + parametric surfaces + NURBS + ..., all stitched together at edges
    • Parasolid, ACIS, ...

Issues in Boundary Representation Solid Modeling

• Very complex data structures
  – NURBS-based winged-edges, etc.
• Complex algorithms
  – manipulation, booleans, collision detection
• Robustness
• Integrity
• Translation
• Features
• Constraints and Parameters

Other Issues: Non-Manifold Solids

• There are cases where you may need to model entities that are not entirely 3D

Cell Decomposition

• Set of primitive cells
• Parameterized
• Often curved
• Compose complex objects by gluing cells together
• Used in finite-element analysis
Spatial Occupancy Enumeration

- Brute force
  - A grid
- Pixels
  - Picture elements
- Voxels
  - Volume elements
- Quadtrees
  - 2D adaptive representation
- Octrees
  - 3D adaptive representation
  - Extension of quadtrees

Brute Force Spatial Occupancy Enumeration

- Impose a 2D/3D grid
  - Like graph paper or sugar cubes
- Identify occupied cells
- Problems
  - High fidelity requires many cells
- "Modified"
  - Partial occupancy

Distance Volume

- Store signed distance to surface at each voxel

Offset Surfaces from Distance Volumes

Quadtree

- Hierarchically represent spatial occupancy
- Tree with four regions
  - NE, NW, SE, SW
  - "dark" if occupied

Quadtree Data Structure

F = full  P = partially full  E = empty
Octree
- 8 octants 3D space
  - Left, Right, Up,
    Down, Front, Back

Boolean Operations on Octrees
- $S \cup T$
- $S \cap T$

Adaptive Distance Fields
- Quadtrees/Octrees that store distances

Applications for Spatial Occupancy Enumeration
- Many different applications
  - GIS
  - Medical
  - Engineering Simulation
  - Volume Rendering
  - Video Gaming
  - Approximating real-world data

Issues with Spatial Occupancy Enumeration
- Approximate
  - Kind of like faceting a surface, discretizing 3D space
  - Operationally, the combinatorics (as opposed to the numerics) can be challenging
  - Not as good for applications wanting exact computation (e.g. tool path programming)

Binary Space Partition Trees (BSP Trees)
- Recursively divide space into subspaces
- Arbitrary orientation and position of planes
- Homogeneous regions are leaves called in/out cells
Statistical Representations

- Store density (material vs. void)
- Statistical description of geometry
- Goal – describe the porosity without storing the geometry information

Stochastic Geometry

- Need some way of converting a solid into some representative statistical form
- From each material voxel, calculate the distance to the nearest voxel that is not material
- Repeat for void voxels
- Store distributions:
  - one for empty space
  - one for material
  - density value

Application: Biological Models

- Bone tissue
- MRI data
- Other biological data
- Solid modeling

Application: Surface Texture

- 1 distribution
- 20 spheres

Application: Surface Texture

MRI scan of left shoulder
Bone matrix from scanned data
Overview

• 3D solid model representations
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  – Boxblobs
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  – Quadrees/octrees
  – Stochastic models

Programming Assignment 5

• Extend PBM to support RGB colors (PPM)
• Read 3 models and assign each a color
• Implement Z-buffer rendering
• Implement front & back cutting planes
  – Only render parts of models between planes
• Implement linear depth-cueing
  – Color = base_color+(z-far)/(near-far)
• Re-use and extend 2D polygon filling

End