Lazy Robots and Traveling Guards

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The TSP is hard.

– Barbie
Define Hamiltonian cycle.

If $f(n)$-approximable, finding a tour on right finds a Hamiltonian cycle on left.
Euclidean TSP

General TSP

Euclidean TSP

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Exact Solutions

• Constant TSP
• Distance metric constraints
• Polygon and line/lines
Our Case: Two Polygons

\[ P \quad Q \]
Our Simple Case: Convex
Shortest Tours (Convex): Proof Sketch

(1) 

(2) 

(3) 

(4) 

(5)
Shortest Tours (Convex): Intersections

Can’t cross itself like this:
Shortest Tours (Convex): Intersections

- No intersections on $P$ vertices
- Intersections on $Q$ vertices
Shortest Tours (Convex): Intersections

$P$

$Q$
Some shortest tour visits each $P$ vertex precisely once.
Shortest Tours (Convex): Orientation

Some shortest tour goes around $P$ in cyclic order
Shortest Tours (Convex): Orientation

If a shortest tour is clockwise on $P$ and counter-clockwise on $Q$, then there is precisely one detour.
Shortest Tours (Convex): Detours

- Think of a shortest tour as a set of detours.
- Might need $\min(|P|, |Q|)$ detours
Shortest Tours (Convex): Detours

Might need $\min(|P|, |Q|)$ detours.
Shortest Tours (Convex): Detours

What we know:

Opposite orientation

Same orientation
Call $d_{i,j}^p$ the detour from $p$ through $i \sim j$. Let $c_{i,j}^p$ be its cost.

Let $c_{i,j}$ and $d_{i,j}$ be minima (over $p \in P$).
Shortest Tours (Convex): Detours

A same-orientation shortest tour goes from one detour to the next.
Shortest Tours (Convex): Shortest Paths

1 2 3
• • •

m

O O O

m

O O O

1 2 3

• • •
Shortest Tours (Convex): Shortest Paths
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Find $m$ shortest paths (vertices mod $m$):

- $1 \sim m$
- $2 \sim 1$
- $3 \sim 2$
- etc.
Shortest Tours (Convex): Done!

Summary so far:

- $m$ candidate same orientation tours
- 1 candidate opposite orientation tour

Pick the minimum over $m + 1$ tours!
Shortest Tours (Convex): Recap

(1) Opposite orientation
(2) Same orientation
(3) $\pi$ $p$ $q$

(4) Opposite orientation
(5) Same orientation

$\begin{array}{c}
(1)
(2)
(3)
(4)
(5)
\end{array}$
Shortest Tours: Non-Convex Nested

- Same ideas
- Visibility more complicated
- Intersections on $P$ vertices as well as $Q$
- Backtracking on $P$ as well as on $Q$
- Cyclic on $P$ other than backtracking
Non-Convex, Non-Nested Polygons
Non-Convex, Non-Nested Polygons
Robots

- Cool
- High Tech
- Good demos

Navigation.
Robots: Building Perimeters
Robots: Channel Navigation
Robots: Channel Navigation
More?

$k > 2$ polygons?

- General ETSP in the limit
- Not clear how to generalize detour technique
- Convex hulls might intersect, so can’t simplify in general
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Questions