Theorem 2.1: There are infinitely many primes.

Proof. Assume that there are finitely many primes, $p_1, p_2, ..., p_n$.

By contradiction.

Many $p_i$... $p_j$... $p_k$...
If infinitely many composite numbers exist, then there is a prime not in \(\mathbb{P}\).

Let \(P = P_1 \times P_2 \times ... \times P_n + 1\). Then \(P\) is not in \(\mathbb{P}\) because it is not divisible by any of the primes in \(\mathbb{P}\).
\[
\frac{\frac{1}{x} - 1}{1 + x + \cdots + x^{n+1}} = x^{n+1} - 1
\]

Proof of Geometric Series

Let \( x \neq 1 \) where \( x \in \mathbb{R} \) and

\[ p(n) \geq 2 \]

Consequence of Theorem 2.1
In particular $1 + 2 + \cdots + 2^n = 2^{n+1} - 1$

Use induction to show

$P_n < 2^n$, true $N = 1$.

Let $P_1, \ldots, P_n$ be the first $N$ primes

$P_{n+1} \leq P_1 \cdots P_n + 1$

$< 2 \cdot 2^n + 1 \geq 2^{n+1} + 1$
\[
\sum_{i=1}^{n} \log_{2} x_i \geq \log_{2} \prod_{i=1}^{n} x_i
\]

For any \( e > 0 \), \( X \geq e \).

\[
\prod_{x \geq e} P(x) \geq \log_{\log e} X
\]

Now let show:

\[
2 + 2 + \ldots + 2 = 2^{N + 1 - 1} \geq 2^{N + 1} - 1 > 2^{N + 1}
\]
This works for $e \leq e^x$, but the theorem holds for $x \geq 2$.

Check $e^2 \leq e^x$, $e^2 = 7.389$.

Note: $\log_e e^3 = 3$. 
\[ \theta + 1, \ldots, \theta + (n+1) \]

So \( \theta + 1 \in \mathbb{Z} \).

\[ \frac{1}{2}, \ldots, \frac{1}{2^n} \]

Then \( \mathbb{Q} \cap (0, 1) \).

\[ \frac{1}{2^n} \]

\[ \mathbb{Q} \]

Proof of \( \mathbb{Q} \cap (0, 1) \).

Let \( \mathbb{Q} \cap (0, 1) \) be a dense set between consecutive primes.

In essence, there are infinitely many primes between every pair of consecutive primes.
\[ p_M(p) = p \mod p^{p-1} \]  

If prime (Mersenne prime) \[ p = \frac{2^n - 1}{2} \]

28 = 1 + 2 + 4 + 7 + 14

6 = 1 + 2 + 3

p = \text{sum of divisors (not including itself)}

p is a perfect number if \( p \) is a prime.
\[ 2^{m} - 1 \equiv (2^{m-1}) \equiv (2^{(m-1) \mod 2} + 1) \]

\[ \{ p \neq 2 \text{ and prime } W(p) \text{ not finite} \} \]

\[ W(p) = 4, t = 2 \]

\[ p = 3, W(p) = 2 \]

\[ p = 5, W(p) = 2 \]

\[ p = 7, W(p) = 3 \]
\[
\sum_{Z_p \in \mathbb{Z}^n} \prod_{p=1}^{n} \mathbb{M}(p) = (Z_p - 1)^{-1} \mathbb{C}^{(Z_p)} \mathbb{C}^{(Z_p)}
\]
It is not known if there are any odd perfect numbers.

In fact, all even perfect numbers are of the form $2^{p-1} \cdot M(p)$.
\[
\frac{2^{n+1} - 1}{2n + 1} = 2^n \quad \text{for} \quad n \geq 0
\]

\[
S = \lim_{n \to \infty} \left( \frac{2 \cdot 2^3}{2^2} \right) = \sum_{n=0}^{\infty} (2^n)
\]

Proof:

\[
S = \sum_{n=1}^{\infty} \frac{2^n}{n+1}
\]

Theorem 2.4.7

Lemma 2.4.6
This must be an integer.

\[
\frac{\text{integer}}{\frac{2a+1}{t}} + \frac{2a+1}{t} + \frac{1}{t} = \frac{1}{t} \left( 2a+1 \right) \frac{1}{t} + \frac{1}{t} \left( 2a+1 \right) \left( 1 - \frac{1}{t} \right) = \frac{2a+1}{t} - 1
\]
\[ \frac{2^{a+1} - 1}{t} = 1 \Rightarrow t = 2^{a+1} - 1 \]

and \( t \) is prime.

Then we have drawn

\[ \text{when } \text{sum } S = 5 \]

\[ t \cdot \frac{2^{a+1} - 1}{t} \text{ can only be } 3, 5 \]