Pseudo Transpose

By the commutative theorem

\[ (W_2 \otimes I_{2^n}) = L_{2^n}^{\min} (I_{2^n} \otimes W_2) L_{2^n}^{\min} \]

However, unless \( m = n \), the stride permutation can not easily be done in place. We show another permutation that also works, but can easily be done in place.

Bit permutations

\[ W_2 \otimes I_4 = L_2^t (I_4 \otimes W_2) L_4^t \]

View \( L_2, L_4 \) as permute of the bits in the binary address of the indices of the input vectors.

\[
\begin{pmatrix}
  x_0 \\
  x_1 \\
  x_2 \\
  x_3 \\
  x_4 \\
  x_5 \\
  x_6 \\
  x_7
\end{pmatrix}
\]

\[
\begin{pmatrix}
  x_0 \\
  x_2 \\
  x_4 \\
  x_6 \\
  x_1 \\
  x_3 \\
  x_5 \\
  x_7
\end{pmatrix}
\]

<table>
<thead>
<tr>
<th>000 \rightarrow 000 = 0</th>
<th>100 \rightarrow 001 = 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>001 \rightarrow 010 = 2</td>
<td>101 \rightarrow 011 = 3</td>
</tr>
<tr>
<td>010 \rightarrow 100 = 4</td>
<td>110 \rightarrow 101 = 5</td>
</tr>
<tr>
<td>011 \rightarrow 110 = 6</td>
<td>111 \rightarrow 111 = 7</td>
</tr>
</tbody>
</table>
\[
\begin{align*}
111 - 101 & = 110 \\
110 - 111 & = 101 \\
110 - 110 & = 010 \\
110 - 101 & = 001 \\
100 - 100 & = 000 \\
\end{align*}
\]

Permutation

2 positions, leads to a selection

When does the book begin our story?

Permutation is called as Inception (Order 2),

we can find which one is suitable. Such a

After that, what is that permutation is a
difficult...

...and look at 

\[
\begin{align*}
111 - 111 & = 0 \\
110 - 110 & = 0 \\
100 - 100 & = 0 \\
\end{align*}
\]

& \log_2 \frac{1}{3}

Conjures a second permutation of the soup

Cycles the story, so far, if the book.

Andre: that is like Internal points to

Interpreting $M_{(0,2)}^{y}$ as a block transposion.

Recall if

$$X = \begin{pmatrix} x_0 & x_1 & x_2 & x_3 \\ x_4 & x_5 & x_6 & x_7 \end{pmatrix}, \quad X^T = \begin{pmatrix} x_0 & x_1 \\ x_2 & x_3 \\ x_4 & x_5 \\ x_6 & x_7 \end{pmatrix}$$

$$p(X) = \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix}, \quad p(X^T) = \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

So,

$$L^T p(X) = p(X^T)$$

Observe

$$\tilde{X} = \begin{pmatrix} x_0 & x_4 & x_7 \\ x_1 & x_5 & x_7 \\ x_2 & x_6 & x_7 \end{pmatrix}$$

"black transposion" or "pseudo transposion."

$$p(\tilde{X}) = \begin{pmatrix} x_0 \\ x_4 \\ x_2 \\ x_4 \\ x_1 \\ x_5 \\ x_3 \\ x_2 \end{pmatrix} = M_{(0,2)}^{y} p(X)$$
It turns out both \( L^g_4 \) and \( M^{(0,2)} \) can be used to commute \( \omega_2 \otimes \Gamma_i \)

First observe

\[
L^g_4 (e_i \otimes e_j \otimes e_k) = e_j \otimes e_k \otimes e_i
\]

\[
L^g_4 e_i \otimes e_j \otimes e_k = e_i \otimes e_j \otimes e_k + e_j \otimes e_i \otimes e_k
\]

This relates the \( \otimes \) to "bit" permutations.

\[
M^{(0,2)} (e_i \otimes e_j \otimes e_k) = e_i \otimes e_j \otimes e_k
\]

Now we can show that the commutation theorem for both permutations.

\[
(W_2 \otimes I_4) (e_i \otimes e_j \otimes e_k) = W_2 e_i \otimes e_j \otimes e_k
\]

\[
L^g_2 (I_4 \otimes W_2) L^g_4 (e_i \otimes e_j \otimes e_k)
\]

\[
= L^g_2 (I_4 \otimes W_2) e_j \otimes e_i \otimes e_k
\]

\[
= L^g_2 e_j \otimes e_i \otimes W_2 e_k
\]

\[
e_i \otimes e_j \otimes e_k
\]

which is the same as above, and since this holds for all basis elements is true in general.
\[ M_{(0,1)}^g (I\gamma \otimes \omega_2) M_{(0,1)} e_i \otimes e_j \otimes e_k \]

\[ = M_{(0,1)}^g (I\gamma \otimes \omega_2) e_k \otimes e_j \otimes e_i \]

\[ = M_{(0,1)}^g e_k \otimes e_j \otimes \omega_2 e_i \]

\[ = \omega_2 e_i \otimes e_j \otimes e_k \]

The permutations \( M_{(0,1)}^g \) and \( L_{\gamma, L^g} \) are examples of bit permutations.

\[ P_\sigma (e_{i_0} \otimes \cdots \otimes e_{i_6}) = e_{\sigma(i_0)} \otimes \cdots \otimes e_{\sigma(i_6)} \]

and any bit permutation with the property

\[ P_\sigma (e_{i_0} \otimes \cdots \otimes e_{i_{m-1}} \otimes e_{i_m} \otimes \cdots \otimes e_{i_{m+n-1}}) \]

\[ = e_{\sigma(i_0)} \otimes \cdots \otimes e_{\sigma(i_{m-1})} \otimes e_{\sigma(i_m)} \otimes \cdots \otimes e_{\sigma(i_{m+n-1})} \]

will satisfy \( \sigma(i) \in \{0, \ldots, m\} \) with \( 0 \leq i \leq m \)

and \( \sigma(i) \in \{0, \ldots, m\} \)

\[ m \leq i \leq m+n \]

\[ P_\sigma^{-1} (I_{2^m} \otimes \omega_{2^n}) P_\sigma = \omega_{2^n} \otimes I_{2^m} \]
It follows the following proposition

\[ M_{\text{pre}} = I \otimes M_{\text{post}} \]

where \( M_{\text{pre}} \) is the pre-

\[ (M_{\text{pre}} (x)) \quad (M_{\text{post}} (y)) \]

Finally, observe that

\[ \text{logic} \leftarrow \text{clar} \]

This column address leads to the pseudo-

\[ \text{the non} \]
\[
\bar{X} = R \left( \begin{array}{c|c|c|c}
\bar{X}_{o_0,0} & \bar{X}_{o_1,1} & \cdots & \bar{X}_{o_n,0} \\
\hline
R & R & \cdots & R \\
\hline
R & R & \cdots & R \\
\end{array} \right) \\
\begin{array}{c}
\bar{X}^T \\
\hline
\bar{X}_{o_0,0}^T & \bar{X}_{o_1,1}^T & \cdots & \bar{X}_{o_n,0}^T \\
\hline
\end{array}
\]

\[
M^{RS} \left( \mathbb{I}_B \otimes W \right)
\]

4) \[
M^{RS}_R \left( I_S \otimes W_R \right) M^{RS}_R \\
= W_R \otimes I_S
\]