Strassen (Winograd) op

$$T(n) = 7T(n/2) + 15(n/2)^2$$

Assume $n = 2^k$.

$$T(n) = 7\left(7T(n/4) + 15(n/4)^2\right) + 15(n/2)^2$$

$$= 7^2 T(n/2^2) + \frac{15}{4}n^2(7/4) + 15/4\, n^2$$

$$= 7^2\left(7T(n/2^2) + 15/4\, n^2(7/4)^2\right) + 15/4\, n^2\left((7/4) + 1\right)$$

$$= 7^3 T(n/2^3) + 15/4\, n^2 \sum_{i=1}^{2} (7/4)^i$$

- - -

$$T(n) = 7^j T(n/2^j) + 15/4\, n^2 \sum_{i=0}^{j-1} (7/4)^i$$

When $j = k$, $n/2^j = 1$

and $T(1) = 1$

$$\Rightarrow T(n) = 7^k + \frac{15}{4} n^2 \sum_{i=0}^{k-1} (7/4)^i$$

$$= 7^k + \frac{15}{4} n^2 \left(\frac{(7/4)^k - 1}{7/4 - 1} \right)$$

$$= 7^k + \frac{15}{4} \cdot \frac{4}{3} n^2 \left((7/4)^k - 1 \right)$$

$$= 7^k + 5 \left(7^k - 4^k \right)$$

$$\left[n^2 = (2^k)^2 = 4^k \right)$$

$= 6 \cdot 7^k - 5 \cdot 4^k$

In terms of n

$= 6n^{\lg(7)} - 5n^2, \quad \lg(7) \approx 2.807$

For Regular matrix mult.

# of ops $= 2n^3 - n^2$

Now assume $n = 2^k c$

with base case

$$T(c) = 2c^3 - c^2$$

In this case

$$T(n) = 7^k T(c) + \frac{15}{4} n^2 \sum_{i=0}^{k-1} (7/4)^i$$

$$= 7^k \left(2c^3 - c^2\right)$$

$$+ 5n^2 \left(\left(7/4\right)^k - 1\right)$$

$$\left[\; 4^k = \left(n/c\right)^2 \;\right]$$

$$= 7^k \left(2c^3 - c^2\right)$$

$$+ 5c^2 \left(7^k - \left(n/c\right)^2\right)$$

$$= 7^k \left(2c^3 - c^2 + 5c^2\right) - 5n^2$$

$$= 7^k \left(2c^3 + 4c^2\right) - 5n^2$$

$$= n^{\lg(7)} \left(2c^3 + 4c^2\right) - 5n^2$$