A Database Framework for Probabilistic Preferences

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Abstract. We propose a framework that naturally represents preferences in a probabilistic relational database. The framework augments the relational schema with a special type of a symbol—a preference symbol. Effectively, an instance of a preference symbol can be internally represented as a collection of parametric preference distributions such as Mallows. We establish positive and negative complexity results for evaluating Conjunctive Queries (CQs) over databases where preferences are represented in the Repeated Insertion Model (RIM), Mallows being a special case. This paper is an abbreviated version of our PODS 2017 paper, where an interested reader can find additional details about the formalism and proposed algorithmic solutions.

1 Introduction

Preferences are statements about the relative quality or desirability of items. Ever larger amounts of preference information are being collected and analyzed in a variety of domains, including recommendation systems [2, 16, 18], polling and election analysis [3, 6, 7, 15], and bioinformatics [1, 11, 19].

Preferences are often inferred from indirect input (e.g., a ranked list may be inferred from individual choices), and are therefore uncertain in nature. This motivates a rich body of work on uncertain preference models in the statistics literature [14]. More recently, the machine learning community has been developing methods for effective modeling and efficient inference over preferences, with the Mallows model [13] receiving particular attention [4, 5, 12, 17].

In this paper, we take the position that preference modeling and analysis should be accommodated within a general-purpose probabilistic database framework. Our framework is based on a deterministic concept that we proposed in a past vision paper [8]. In the present work we focus on handing uncertain preferences, and develop a representation of preferences within a *probabilistic preference database*, or *PPD* for short.

2 Probabilistic Preference Databases

A *preference schema* **S** is a relational schema with some relation symbols marked as *preference symbols* (and others as *ordinary symbols*). Figure 1 gives an example of a preference database instance, with the ordinary symbols **Candidates** and **Voters**, and the preference symbol **Polls**.

Candidates (o)

cand	party	sex	edu
Trump	R	M	BS
Clinton	D	F	JD
Sanders	D	M	BS
Rubio	R	M	JD

Voters (o)

voter	edu	sex	age
Ann	BS	F	25
Bob	BS	M	35
Cat	MS	F	40
Dave	MS	M	45

Polls (p)

voter	date	lcand	rcand
Ann	Oct-5	Sanders	Clinton
Ann	Oct-5	Sanders	Rubio
Ann	Oct-5	Sanders	Trump
Ann	Oct-5	Clinton	Rubio
Ann	Oct-5	Clinton	Trump
Ann	Oct-5	Rubio	Trump
Bob	Oct-5	Sanders	Rubio
Bob	Oct-5	Sanders	Clinton
Bob	Oct-5	Sanders	Trump
Bob	Oct-5	Rubio	Clinton
Bob	Oct-5	Rubio	Trump
Bob	Oct-5	Clinton	Trump

A **MAL**-instance over **Polls** (p)

voter	date	*Preference model* $\mathbf{MAL}(\boldsymbol{\sigma}, \phi)$
Ann	Oct-5	$\langle$Clinton, Sanders, Rubio, Trump$\rangle$, 0.3
Bob	Oct-5	$\langle$Trump, Rubio, Sanders, Clinton$\rangle$, 0.3

Fig. 1. An example of a preference database

An instance over a preference symbol (such as **Polls** in Figure 1) represents a collection of *preferences* among a set of items, where each such preference is itself a binary relation called a *session*. A a binary relation $\succ$ over a set $I = \{\sigma_1, \ldots, \sigma_n\}$ of *items* is a (strict) *partial order* if it is irreflexive and transitive. A *linear* (or *total*) order is a partial order where every two items are comparable. By a slight abuse of notation, we often identify a linear order $\sigma_1 \succ \cdots \succ \sigma_n$ with the sequence $\langle \sigma_1, \ldots, \sigma_n \rangle$, and we call it a *ranking*.

Example 1. Our running example is on individual preferences among the set of US presidential candidates $I = \{\texttt{Clinton}, \texttt{Rubio}, \texttt{Sanders}, \texttt{Trump}\}$. The ranking $\boldsymbol{\tau} = \langle \texttt{Clinton}, \texttt{Rubio}, \texttt{Sanders}, \texttt{Trump} \rangle$ is an example ranking over I. □

A preference relation instantiates a special relation symbol with a signature of the form (β, A_l, A_r), where β is the *session signature*, and A_l and A_r are the *left-hand-side* (*lhs*) attribute and *right-hand-side* (*rhs*) attribute, respectively. We use semicolon (;) to distinguish between the different parts and write $(\beta; A_l; A_r)$.

Example 2. We use the preference signature (voter, date; lcand; rcand) in our running example. Here the components β, A_l and A_r are (voter, date), lcand, and rcand, respectively. The table **Polls** of Figure 1 is an instance of this preference signature that contains two sessions. The session (Ann, Oct-5) is associated with the ranking $\langle$Sanders, Clinton, Rubio, Trump$\rangle$. The tuple (Ann, Oct-5; Sanders; Clinton) denotes that in the session of the voter Ann on October 5th, the candidate Sanders is preferred to the candidate Clinton. □

We now make the knowledge about voters' opinions probabilistic, interpreting the preference database of Figure 1 as one possible world of a probabilistic preference database. A *probabilistic preference database* (abbrv. *PPD*) over the preference schema **S** is a probability space over preference databases over **S**. A PPD can be represented by explicitly specifying the entire sample space; however, we wish to allow for standard compact representations of preferences.

A *probabilistic preference model* is a (finite and typically compact) representation M of a probability space over partial orders $\succ$ over a finite set of items; we denote this probability space by $[\![M]\!]$. A *model family* is a collection $\mathcal{M}$ of probabilistic preference models. As prominent examples, we define two model families: **RIM** is the family of RIM [5] models $\mathsf{RIM}(\boldsymbol{\sigma}, \Pi)$, and **MAL** is the family of Mallows [13] models $\mathsf{MAL}(\boldsymbol{\sigma}, \phi)$.

A *Mallows* model [13] $\mathsf{MAL}(\boldsymbol{\sigma}, \phi)$ is parameterized by a reference ranking $\boldsymbol{\sigma} = \langle \sigma_1, \ldots, \sigma_m \rangle$ and a dispersion parameter $\phi \in (0, 1]$. The model assigns a non-zero probability to every ranking $\boldsymbol{\tau}$: The higher the Kendal's tau distance [9] of $\boldsymbol{\tau}$ is from $\boldsymbol{\sigma}$, the lower its probability under the model. Lower values of ϕ concentrate most of the probability mass around $\boldsymbol{\sigma}$, while $\phi = 1$ corresponds to the uniform probability distribution over the rankings. Doignon [5] showed that $\mathsf{MAL}(\boldsymbol{\sigma}, \phi)$ can be represented as the insertion model $\mathsf{RIM}(\boldsymbol{\sigma}, \Pi)$.

In the PPD representations we explore, termed **RIM**-PPD, each session is associated with the parameters of a RIM model. A **RIM**-PPD represents a probability space over preference databases, where a possible world is obtained by independently sampling a preference from the model of each session. Figure 1 gives an example of a **MAL**-instance over the p-symbol **Polls** that associates each session in **Polls** with a Mallows model.

3 Query Evaluation over PPDs

We adopt the semantics of probabilistic databases [20] for query evaluation. Specifically, let $\mathbf{S}$ be a schema, let Q be a query, and let $\mathcal{D} = (\Omega, \pi)$ be a PPD. A *possible answer* for Q is a tuple $\mathbf{a}$ over $sig(Q)$ such that $\mathbf{a} \in Q(D)$ for some sample D of $\mathcal{D}$. We denote by $PosAns(Q, \mathcal{D})$ the set of all possible answers. The *confidence* of a possible answer $\mathbf{a} \in PosAns(Q, \mathcal{D})$, denoted $conf_Q(\mathcal{D}, \mathbf{a})$, is the probability of having $\mathbf{a}$ as an answer when querying a sample of $\mathcal{D}$. If E is an $\mathcal{M}$-PPD for some model class $\mathcal{M}$, then evaluating Q on E is the task of computing the following (finite) set: $Q(E) = \{(\mathbf{a}, conf_Q([\![E]\!], \mathbf{a})) \mid \mathbf{a} \in PosAns([\![E]\!])\}$.

We study the data complexity of evaluating Conjunctive Queries (CQs) over **RIM**-PPDs. We focus on CQs to which we refer as *itemwise*. Intuitively, these are CQs where items are connected only through preferences. We show a natural fragment of CQs where the itemwise CQs are *precisely* the CQs in which query evaluation can be done in polynomial time. In the fragment we consider, we prove that every query that is *not* itemwise is actually #P-hard, and therefore, we establish a dichotomy in complexity.

Let $\mathbf{S}$ be a preference schema, and let Q be a CQ over $\mathbf{S}$. An atomic formula of Q is called a *p-atom* if it is over a p-symbol, and an *o-atom* if it is over an o-symbol. Let $P(s_1, \ldots, s_k; t_l; t_r)$ be p-atom of Q. Each term s_i for $i = 1, \ldots, k$ is said to occur in a *session position*, and each of t_l and t_r is said to occur in an *item position*. A *session variable* of Q is a variable that occurs in a session position, and an *item variable* of Q is a variable that occurs in an item position. We say that Q is *sessionwise* if all p-atoms of Q refer to the same session; that is, if $P(s_1, \ldots, s_k; t_l; t_r)$ and $P'(s'_1, \ldots, s'_l; t'_l; t'_r)$ are p-atoms of Q, then $P = P'$

and $(s_1, \ldots, s_k) = (s'_1, \ldots, s'_l)$. We say that Q is *itemwise* if Q is sessionwise, and the joins between item variables occur only inside the p-atoms, or through session variables. Put differently, in an itemwise CQ with a constant session, the o-atoms state properties of individual items but do not draw connections between the items. In [10] we define this property more formally, by means of the *Gaifman graph* of the CQ.

Example 3. Consider the following Boolean CQs. The query Q_1 asks whether there is a voter with a BS degree who prefers a male Democratic candidate to a female Democratic candidate.

$$Q_1() \leftarrow P(v, \_; l; r), V(v, \texttt{BS}, \_, \_), C(l, \texttt{D}, \texttt{M}, \_), C(r, \texttt{D}, \texttt{F}, \_)$$

The query Q_2 asks whether there is a voter who prefers a male candidate to a female candidate such that both candidates are of the same political party.

$$Q_2() \leftarrow P(\_, \_; l; r), C(l, p, \texttt{M}, \_), C(r, p, \texttt{F}, \_)$$

The query Q_3 asks whether there is a voter who prefers a female candidate to both Trump and Sanders.

$$Q_3() \leftarrow P(v, d; l; \texttt{Trump}), P(v, d; l; \texttt{Sanders}), C(l, \_, \texttt{F}, \_)$$

All of these CQs are sessionwise. Indeed, Q_1 and Q_2 involve a single p-atom (hence, they are sessionwise by definition), and in Q_3 both atoms have (v, d) in their session parts. CQs Q_1 and Q_3 are itemwise, while Q_2 is *not* itemwise. □

In [10] we prove the following theorem, which states that every itemwise Boolean CQ can be evaluated in polynomial time, under data complexity.

Theorem 1. *Let* $\mathbf{S}$ *be a preference schema, and let* Q *be a Boolean CQ over* $\mathbf{S}$*. If* Q *is itemwise, then* Q *can be evaluated in polynomial time on* **RIM***-PPDs.*

We also prove that the class itemwise CQs are *precisely* the tractable ones (among the queries in the class), under conventional complexity assumptions. In other words, every Boolean CQ (in the class) that is not itemwise is necessarily hard to evaluate, and therefore, we establish a dichotomy.

Theorem 2. *Let* $\mathbf{S}$ *be a preference schema, and let* Q *be a Boolean CQ over* $\mathbf{S}$ *such that* Q *has no self joins and* Q *has a single p-atom. If* Q *is not itemwise, then the evaluation of* Q *on* **RIM***-PPDs over* $\mathbf{S}$ *is* $\mathrm{FP}^{\#\mathrm{P}}$*-hard.*

In [10] we give is a polynomial-time algorithm for evaluating itemwise CQs. Interestingly, such CQs translate into a natural (and novel) inference problem over RIM. In this problem, every item is associated with one or more labels (e.g., "democratic" party or "comedy" genre), and the goal is to compute the probability that a graph pattern (or equivalently a partial order) over these labels *matches* the random ranking.

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