Review 3

1. Express NAND and the equivalence in terms of the implication.

2. Apply laws of propositional logic to transform the expression

$$(p \to q)(q \to r) \to (p \to r)$$

into constant 1. Justify each step of your transformation.

3. Are conditions
(i) $4|n$,
(ii) $4|n^2$,
where n is a positive integer larger than 10, equivalent? Provide either a proof or a counterexample.

4. Take expressions $p \to q$ and $qr \to s$ as hypotheses and prove the formula $pr \to s$ by deduction. You are allowed to use only the rules of inference provided below.
(a) Any tautology may be used as a line of proof.
(b) If E and $E \to F$ are lines of a proof, then F may be added as a line of the proof.
(c) If E and F are lines of a proof, then $E\,F$ may be added as a line of the proof.
(d) If E and $E \equiv F$ are lines of a proof, then F may be added as a line of the proof.

5. Write down the expression $(p \to q)(r \to s)$ in both conjunctive and disjunctive normal forms.

6. Let E be the expression given below.

$$p(X, Y) \to (\exists Z)\,(p(X, Z)\ \text{AND}\ p(Z, Y))$$

(i) Draw the expression tree of E.
(ii) Take positive integers as the domain and the relation $U|V$ as the interpretation for predicate $p(U, V)$. Is the interpretation true?
(iii) Take positive integers as the domain and the relation $U|V$ and $U \neq V$ as the interpretation for predicate $p(U, V)$. Is the interpretation true?

7. Place an appropriate logical operator $\to$,$\leftarrow$, or $\equiv$ instead of the symbol ? so that the resulting expression becomes a tautology of predicate calculus.

$$(\exists X)\,(p(X)\ \text{AND}\ q(X))\ ?\ ((\exists X)\,p(X)\ \text{AND}\ \ (\exists X)\,q(X))$$

Provide a justification of your answer.

8. What is the maximal number of edges of the expression tree that one needs to cross in order to reach the bounding quantifier corresponding to a given variable? Can it happen that this maximal number equals to the height of the expression tree? Can it happen that it equals to the number of quantifiers occurring inside the expression tree? Provide examples justifying your answers.