Section 4.3 Amortization and Sinking Funds

Amortization: The gradual elimination of a liability, such as a mortgage, in regular payments over a specified period of time. Such payments must be sufficient to cover both principal and interest.

Amortization Formula: The periodic payment $R$ on a loan of $P$ dollars to be amortized over $n$ periods with interest charged at the rate of $i$ per period is

$$R = \frac{Pi}{1 - (1+i)^{-n}}$$

But again we can use the formula:

$$P(1 + i)^n = R\left(\frac{(1 + i)^n - 1}{i}\right)$$
Example 1: A sum of $50,000 is to be repaid over a 5-year period through equal installments made at the end of each year. If an interest rate of 8% per year is charged on the unpaid balance and interest calculations are made at the end of each year, determine the size of each installment so that the loan is amortized at the end of 5 years.

Solution: use

\[ P(1 + i)^n = R \left( \frac{(1 + i)^n - 1}{i} \right) \]

Find \( R \), given that \( P=50,000 \) \( r=.08 \) \( m=1 \) \( i=.08 \) \( n=5 \)

\[ R = 12,522.82 \]
# An Amortization Schedule

<table>
<thead>
<tr>
<th>End of Period</th>
<th>Interest Charged</th>
<th>Repayment Made</th>
<th>Payment Toward Principle</th>
<th>Outstanding Principal</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$4,000.00</td>
<td>$12,522.82</td>
<td>$8,522.82</td>
<td>$50,000.00</td>
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<td>12,522.82</td>
<td>11,595.21</td>
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<tr>
<td>5</td>
<td>--</td>
<td>12,522.82</td>
<td>12,522.82</td>
<td>0.01</td>
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</table>
Example 2: Home Mortgage Payments. The Blakelys borrowed $120,000 from a bank to help finance the purchase of a house. The bank charges interest at a rate of 9% per year on the unpaid balance, with interest computations made at the end of each month. The Blakelys have agreed to pay the loan in equal monthly installments over 30 years. How much should each payment be if the loan is to be amortized at the end of the term?

Solution: use

\[ P(1 + i)^n = R \left( \frac{(1 + i)^n - 1}{i} \right) \]

Find \( R \), given that \( P=120,000 \quad r=.09 \quad m=12 \quad i=.09/12=.0075 \)
\( n=(30)(12)=360 \)

\[ R=965.55 \]
Equity is equal to total assets minus liabilities.

In other words, how much something is worth minus the present value of how much you still have to pay.
Example 3: Home Equity. Teresa and Raul purchased a house 10 years ago for $200,000. They made a down payment of 20% of the purchase price and secured a 30-year conventional home mortgage at 9% per year on the unpaid balance. The house is now worth $280,000. How much equity do Teresa and Raul have in their house now (after making 120 monthly payments)?

Solution: The down payment was 20% of 200,000 which is 40,000. The loan was for the remaining $160,000.

use: 
\[ P(1 + i)^n = R \left( \frac{(1 + i)^n - 1}{i} \right) \]  

Find \( R \), given that \( P=160,000 \quad r=.09 \quad m=12 \quad i=.09/12=.0075 \)  
\( n=(30)(12) \)

\[ R=1287.40 \]
(cont.) The house was purchased 10 years ago so 120 payments have been made, the outstanding principal is given by the sum of the present values of the remaining installments (360-120=240 installments). But this sum is just the present value of an annuity with \( n=240 \), \( R=1287.40 \), and \( i=.0075 \).

We can now use the same formula (*) to solve for \( P \).

\[
P=143.088.01
\]

Therefore, Teresa and Raul have an equity of 280,000-143,088, or approximately, $136,912.
Example 4: Home Affordability. The Jacksons have determined that after making a down payment they could afford at most $1500 for a monthly house payment. The bank charges interest at the rate of 7.2% per year on the unpaid balance, with interest computations made at the end of each month. If the loan is to be amortized in equal monthly installments over 30 years, what is the maximum amount that the Jacksons can borrow from the bank?

Solution: Find $P$, given that $R=1500 \quad r=.072 \quad m=12 \quad i=.072/12=.006$

$n=(30)(12)$

use: $P(1+i)^n = R\left(\frac{(1+i)^n - 1}{i}\right)$

$P=220,982$.

So, the Jacksons can borrow at most $220,982$. 
Sinking Funds

A **sinking fund** is an account that is set up for a specific purpose at some future date.

For example, a corporation might establish a sinking fund in order to accumulate capital to replace equipment that is expected to be obsolete at some future date.

We will think of the amount to be accumulated by a specific date in the future as the future value of an annuity \((S)\).

So if we want to find the amount to be accumulated by a specific date in the future we can use:

\[
S = R \left( \frac{(1 + i)^n - 1}{i} \right)
\]
Example 5: Sinking Fund. The proprietor of Carson Hardware has decided to set up a sinking fund for the purpose of purchasing a truck in 2 years’ time. It is expected that the truck will cost $30,000. If the fund earns 10% interest per year compounded quarterly, determine the size of each (equal) quarterly installment the proprietor should pay into the fund.

Solution: We want to find the size of each quarterly payment, \( R \), of an annuity given that its future value is \( S=30,000 \).

\[
S = R \left( \frac{(1 + i)^n - 1}{i} \right)
\]

use:

\[
r = 0.10 \quad m = 4 \quad i = \frac{0.10}{4} = 0.025
\]

\[
R = 3434.02
\]
Sinking Fund Schedule

<table>
<thead>
<tr>
<th>End of Period</th>
<th>Deposit Made</th>
<th>Interest Earned</th>
<th>Addition to Fund</th>
<th>Accumulated Amount in Fund</th>
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<td>647.95</td>
<td>4,081.97</td>
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</table>
Page 248 (19) **Savings Accounts.** Kim invested a sum of money 4 yrs ago in a savings account that has since paid interest at the rate of 6.5%/year compounded monthly. Her investment is now worth $19,440.31. How much did she originally invest?
College Savings Program. The Blakes have decided to start a monthly savings program in order to provide for their son’s college education. How much should they deposit at the end of each month in a savings account earning interest at the rate of 8%/year compounded monthly so that at the end of the tenth year the accumulated amount will be $40,000?
Retirement Accounts. Lee has contributed $200 at the end of each month into her company’s employee retirement account for the past 10 yrs. Her employer has matched her contribution each month. If the account has earned interest at the rate of 8%/year compounded monthly over the 10-yr period, determine how much Lee now has in her retirement account.
Installment Financing. Peggy made a down payment of $400 toward the purchase of new furniture. To pay the balance of the purchase price, she has secured a loan from her bank at 12%/year compounded monthly. Under the terms of her finance agreement, she is required to make payments of $75.32 at the end of each month for 24 mo. What was the purchase price of the furniture?